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TRANSIENT DRAINAGE FROM A VERTICAL SOIL COLUMN[1]

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ABSTRACT

In the soil drainage process, the water content distribution in the unsaturate zone is approximated by a succession of equilibrium water content distributions. Based on this assumption and use of equations for transient drainage, appropriate equations to estimate the cumulative drainage of water from a vertical soil column and the corresponding time requirement have been presented. Agreement between the results of cumulative drainage flow from two different soil columns and those of the analytical equations is quite good when the rate of drainage is small relative to saturated hydraulic conductivity.

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زهکشی یک ستون خاک درحالت ناپایدار

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خلاصه

درحین زهکشی خاک میتوان فرض کردکه درهرلحظه،توزیع رطوبت خاک درمنطقه غیراشباع بالای سطح آب زیرزمینی با توزیع رطوبت خاک تعادلی همان منطقه (وقتیکه زهکشی انجام نمیشود)قابل بیان است .براساس فرض فوق وبابکارگیری معادلات جریان ناپایدار یک بعدی،معادلات لازم برای محاسبه حجم آب خارج شده از یک ستون خاک وزمان لازم برای جمع آوری این حجم آب استخراج ،ودراین مقاله ارائه شده است .مقایسه جوابهای معادلات حاصل با نتایج بدست آمده از زهکشی دوستون خاک متفاوت درآزمایشگاه نشان میدهدکه درحالت کوچک بودن نسبت سرعت زهکشی به هدایت هیدرولیکی درحالت اشباع ،توافق قابل قبولی بین نتایج تجربی وتئوری وجوددارد.

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INTRODUCTION

Solutions for transient drainage from a soil column have been the subject of several studies. These solutions are of two types; 1) numerical solutions, and 2) approximate analytical solutions. In many situations, lack of required information makes numerical solutions inconvenient. Analytical solutions have dealt with idealized soil water systems which might not adequately represent the field.

This paper presents analytical equations for the transient drainage from a soil column which takes the unsaturated region and height of the capillary fringe into account during unsteady drainage. The solutions are based on the assumption that the water content distribution in the partially saturated zone during transient drainage could be approximated by a succession of equilibrium water content distributions. This assumption was used by McWhorter and Duke (10) in their solution for parallel subsurface drains. Experimental investigations were conducted to support the validity of this assumption. The comparison of the scaled laboratory data with the predicted scaled values are presented and discussed.

THEORY FOR ONE-DIMENSIONAL VERTICAL DRAINAGE

Figure 1 shows a column of length L filled with a homogeneous porous medium. The origin of the vertical ordinate Z (positive downward) is the top of the column. The ordinate of the top of the capillary fringe (the plane separating the partially and fully saturated regions) is denoted by $Z_b(t)$ which is a function of time t [T]. Richard's equation

$$\frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] - \frac{dK(\theta)}{d\theta}\frac{\partial \theta}{\partial z} = \frac{\partial \theta}{\partial t} \qquad (1)$$

in which $\theta(zt)$ = volumetric water content; $D(\theta)$ = soil moisture diffusivity $[L^2T^{-1}]$; $K(\theta)$ = effective conductivity

[LT^{-1}]; can in this diffusion form be applied to the unsaturated region (i.e., the region $z \leq z_b$) and, in the saturated zone ($z_b \leq z \leq L$), Darcy's law can be used.

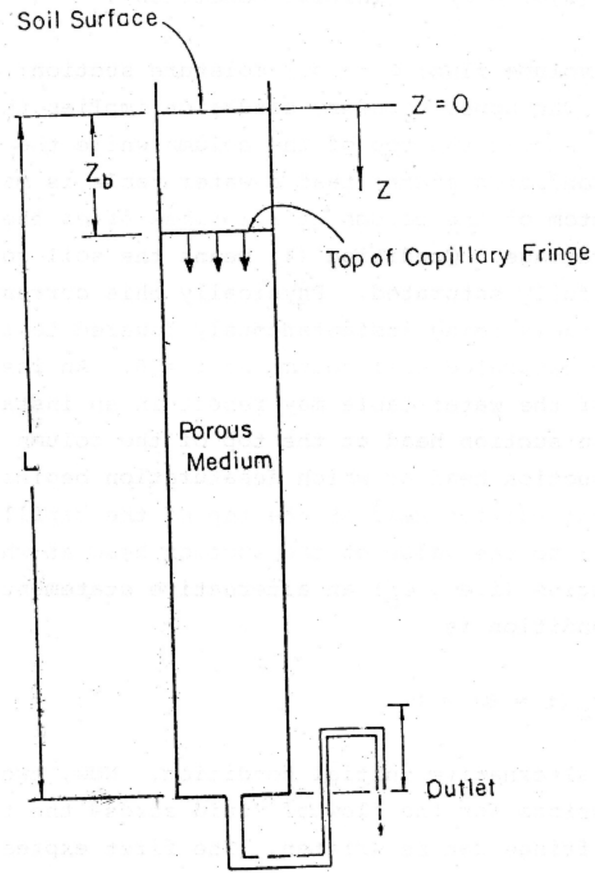

Fig. 1 Schematic sketch of a draining column of porous medium with no recharge.

The boundary and initial conditions are:

$$q(0,t) = 0 \quad \text{(upper boundary condition)} \tag{2}$$

$$\psi(L,t) = 0 \quad \text{(lower boundary condition)} \tag{3}$$

$$\text{and } \theta(z,0) = \phi \quad \text{(initial condition)} \tag{4}$$

where q = volume flux; ψ = soil moisture suction; and ϕ = porosity. The upper boundary condition implies that there is no flow across the top of the column while the lower boundary condition states that a water table is maintained at the bottom of the column at all times after the initiation of drainage. Condition (4) means the soil column is initially fully saturated. Physically this corresponds to the water table being instantaneously lowered to the bottom of a fully saturated soil column at $t = 0$. An instantaneous lowering of the water table may result in an instantaneous increase in suction head at the top of the column to the value of suction head at which desaturation begins $\psi = \psi_b$. Assuming the suction head at the top of the capillary fringe to be equal to the value of the suction head at which desaturation begins (i.e., ψ_b) an alternative statement of the initial condition is

$$z_b(t = 0) = 0 \tag{5}$$

giving an alternative initial condition. Now, two independent expressions for the flow of fluid across the top of the capillary fringe can be written. The first expression results from integration of equation (1) with respect to z over the depth interval $0 \leq z \leq z_b$. Integrating equation (1) subject to the upper boundary condition and letting q_b represent the volume flux density at the top of the capillary fringe, we obtain

$$q_b = \frac{\partial}{\partial t}\left[\int_0^{z_b} \theta dz \right] \tag{6}$$

since at the top of the capillary fringe $\theta(z_b,t) = \phi$ (i.e. saturation), and using the Leibnitz' rule for interchanging the order of integration and differentiaion on the right hand side of equation (6), we get

$$q_b = -\frac{d}{dt}\left[\int_0^{z_b} \theta dz\right] + \phi \frac{dz_b}{dt} \qquad (7)$$

According to Brooks and Corey (1) the equilibrium distribution of water content resulting from drainage in the partially saturated region is

$$\frac{\theta-\theta_r}{\phi-\theta_r} = \left(\frac{z_b-z+\psi_b}{\psi_b}\right)^{-\lambda} \qquad (8)$$

in which θ_r = non-drainable water content; λ is a parameter called the pore-size distribution index by Brooks and Corey (1); and other parameters are defined as before.

Solving equation (8) for θ, substituting the result into equation (7), carrying out the relevant integrations, and then using the scaled variable, $\hat{z}_b = z_b/\psi_b$, we arrive at the equation

$$q_b = (\phi-\theta_r)\psi_b \left\{ \frac{1}{1-\lambda}\frac{d}{dt}\left[1 - (\hat{z}_b+1)^{1-\lambda}\right] + \frac{d\hat{z}_b}{dt} \right\}$$

or

$$q_b = (\phi-\theta_r)\psi_b\left[1 - (\hat{z}_b+1)^{-\lambda}\right]\frac{d\hat{z}_b}{dt} \qquad (9)$$

Separating variables, and integrating subject to the initial condition $z_b(0) = 0$, equation (9) yields

$$\frac{Q(t)}{(\phi-\theta_r)\psi_b} = \hat{z}_b - \frac{(\hat{z}_b+1)^{1-\lambda}}{1-\lambda} + \frac{1}{1-\lambda} \qquad (10)$$

where $Q(t) = q_b t$ and is cumulative drainage flux density.

The second expression for q_b is Darcy's law in the saturated zone ($z_b \leq z \leq L$), which is

$$q_b = K_s \left(\frac{L-z_b-\psi_b}{L-z_b} \right) \tag{11}$$

Equation (11) in its scaled form is

$$q_b = K_s \left(\frac{\hat{L}-\hat{z}_b-1}{\hat{L}-\hat{z}_b} \right) \tag{12}$$

where K_s = saturated hydraulic conductivity and $\hat{L} = L/\psi_b$. Equating the right hand side of equations (9) and (12) and rearranging the result yields

$$\frac{K_s dt}{(\phi-\theta_r)\psi_b} = \frac{\hat{L}-\hat{z}_b}{\hat{L}-\hat{z}_b-1} [1 - (\hat{z}_b+1)^{-\lambda}] d\hat{z}_b \tag{13}$$

Integrating equation (13) over the distance, $0 \leq x \leq z_b$ where x is a dummy variable and time, $t \geq 0$ results in

$$\frac{K_s t}{(\phi-\theta_r)\psi_b} = \int_0^{\hat{z}_b} \frac{\hat{L}-x}{\hat{L}-x-1} [1 - (x+1)^{-\lambda}] dx \tag{14}$$

To solve the integral in equation (14), a numerical or graphical method can be used. Equations (10) and (14) can estimate the cumulative drainage and corresponding time requirement respectively, since other parameters in the equations are constant and have to be evaluated before hand.

It is convenient to scale cumulative discharge, Q, and time, t, by the following relations,

$$\hat{Q} = Q/[\psi_b(\phi-\theta_r)] \tag{15}$$

$$\hat{t} = \hat{K}_s t/[\psi_b(\phi-\theta_r)] \quad (16)$$

The caret symbol, ^, refers to scaled variables throughout this paper.

EXPERIMENTAL INVESTIGATION

The experimental investigation was made on two columns of sand. First, an experiment was conducted on each column to obtain the hydraulic parameters required to describe the hydraulic properties. Secondly, the same columns were resaturated and drainage experiments were performed. Finally, the soil columns were cut into cylindrical segments (20 mm in length) to determine the residual water content. In all experiments the liquid used was water from the domestic water supply that was subsequently de-aired. During the experiments the maximum variations in temperature observed was approximately 0.6°C and thus there was essentially no change in properties of the water during a particular run.

The soil column was constructed from cylindrical lucite sections, 3.2 cm inside diameter. Figure 2 shows the assembly of these sections. Two tensiometer rings were used to measure hydraulic heads at both ends of the test section. PORVIC annular rings were cemented into the groove (approximately 1.6 cm wide) inside each tensiometer ring. Section f, with an annular strip of brass screen cemented into its inside, was used to permit air into the column when required during the experiments.

A graded filter was placed in the bottom of the column. The sand was packed manually. The packing was accomplished by allowing the sand to pass through a nozzle, 2 mm inside diameter, connected to the bottom of the supply funnel, filled with air-dried sand (9).

Before each experimental run, the entire soil column was vacuum saturated. It is not necessary to explain here the procedures used to determine the hydraulic properties of the

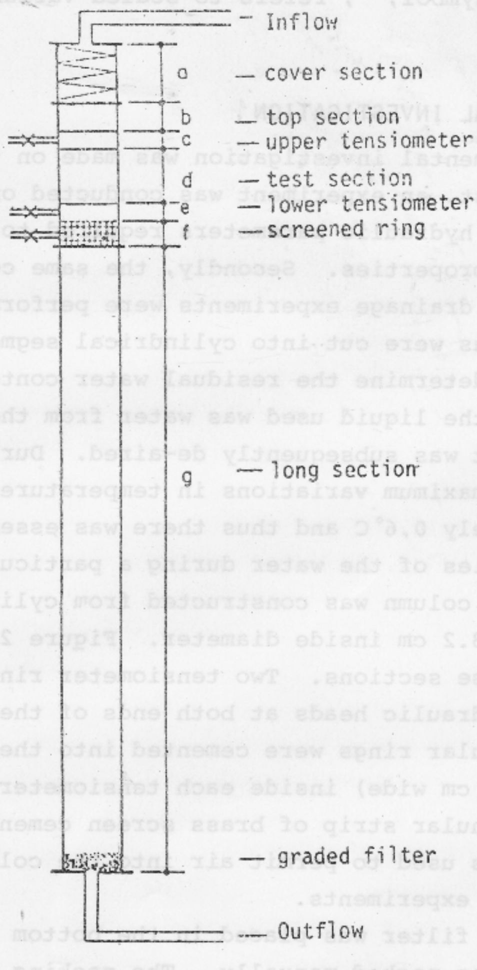

Fig. 2. Sketch of apparatus used for saturation.

sands given in Table 1. These procedures are given in detail by Mahmoodian-Shooshtari (9).

Table 1. Hydraulic Properties of the two sands used.

Parameters	Poudre sand	Ramah sand
K_s	10.08×10^{-2} mm/sec	5.95×10^{-2} mm/sec
ψ_b	520 mm	669 mm
λ	6.4	7.6
ϕ	0.418	0.409
θ_r	0.050	0.091

To obtain unsteady drainage data, two columns were drained. The first column (Poudre sand) drained for a total time of 36 hr, while the second column (Ramah sand) drained for 36.8 hr. The total length of the columns without cover sections was 88.3 and 88.0 cm, respectively. Cumulative outflow measurements versus time were made by passing the outflow into a covered beaker on an electronic balance. Readings were taken at variable time intervals until the discharge was essentially zero.

RESULTS AND DISCUSSION

For each of the sands in Table 1, the cumulative drainage as a function of time was predicted from equations (10) and (14). The scaled theoretical (solid curve) and experimental (discrete points) discharges are plotted against scaled time in

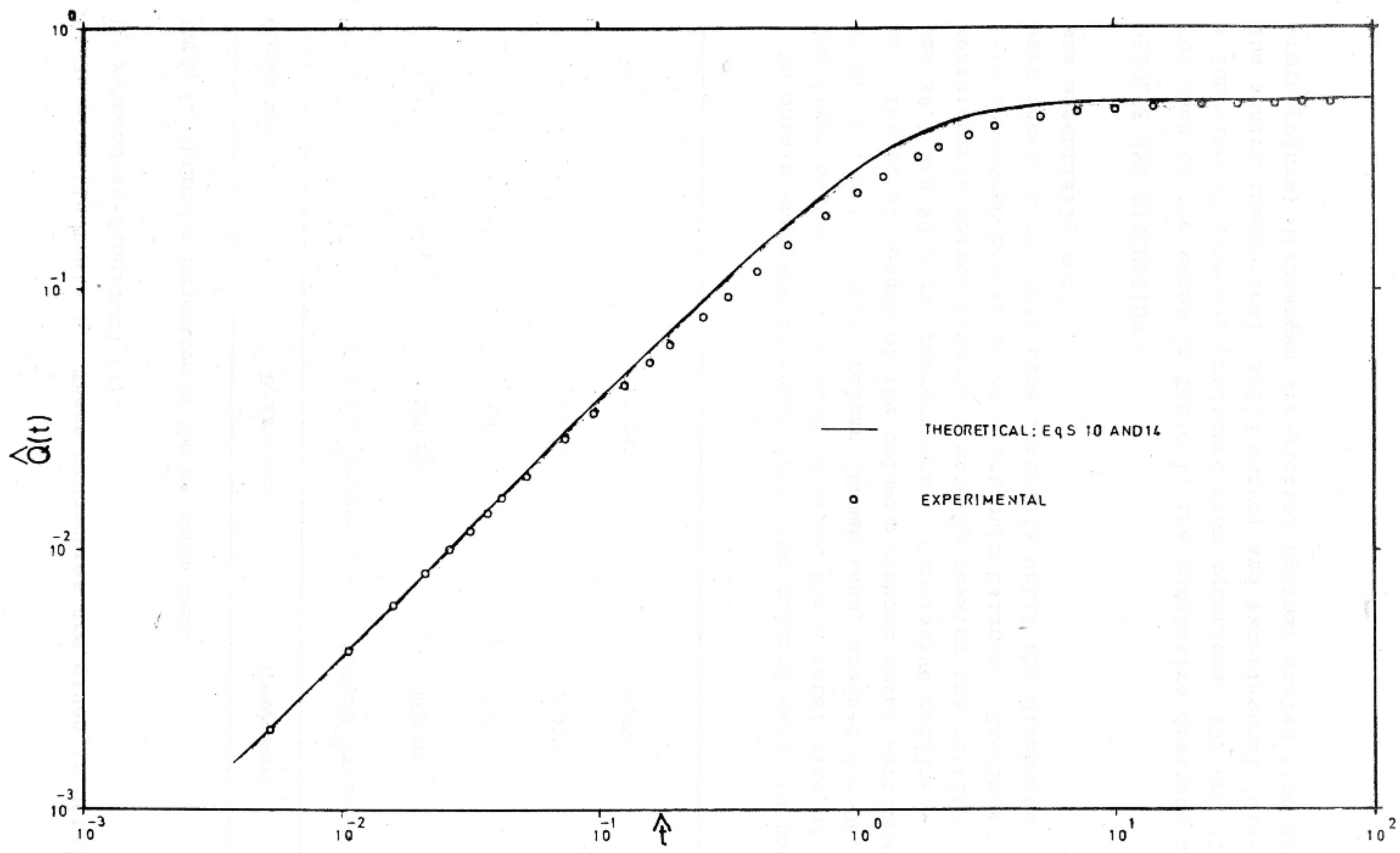

Fig. 3. Scaled discharge-time relationship of unsteady drainage from a column of Poudre sand and theoretical curves.

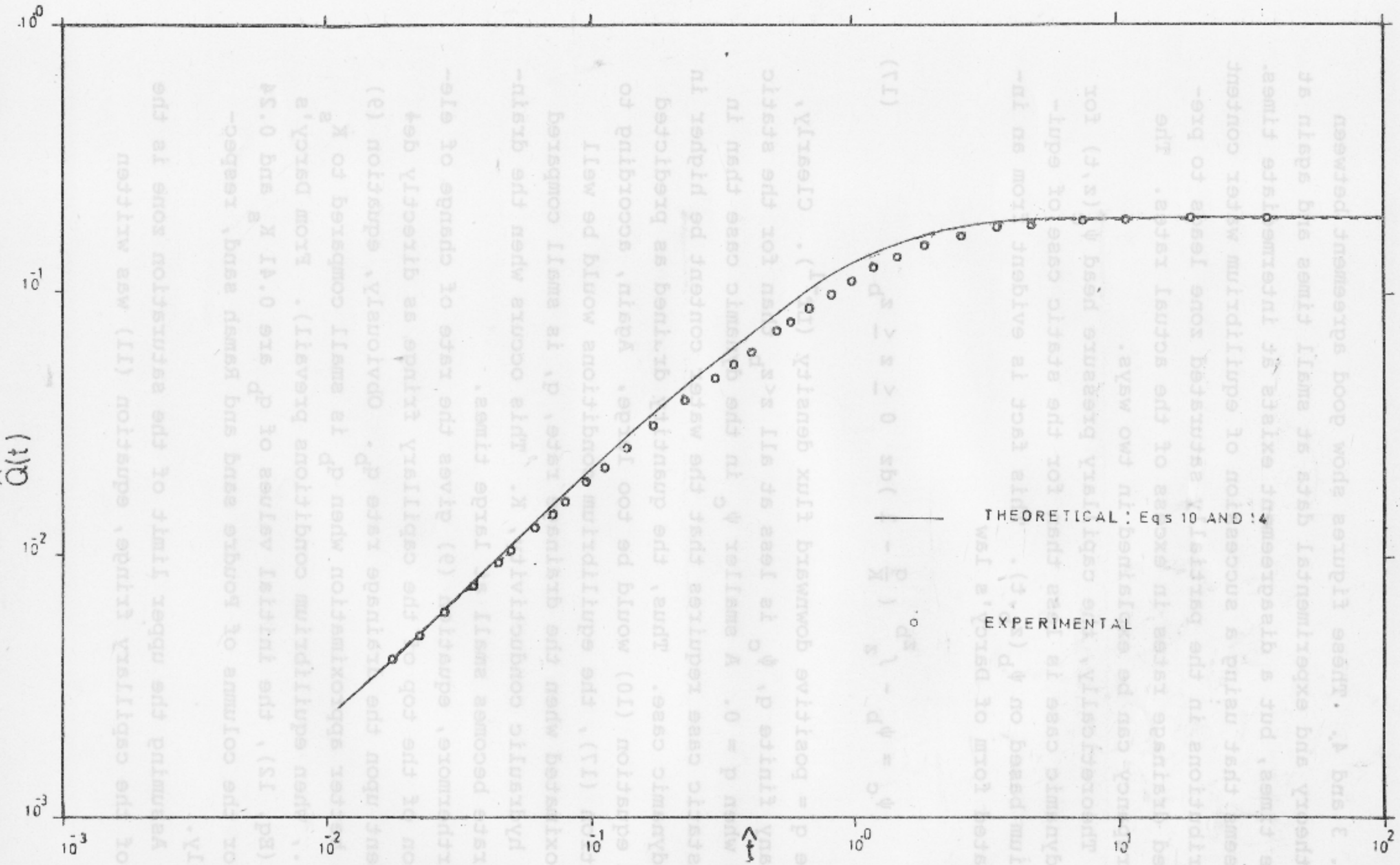

Fig. 4. Scaled discharge time relationship of unsteady drainage from column of Ramah sand and theoretical curves.

Figs. 3 and 4. These figures show good agreement between the theory and experimental data at small times and again at large times, but a disagreement exists at intermediate times. It seems that using a succession of equilibrium water content distributions in the partially saturated zone leads to predicted drainage rates in excess of the actual rates. The discrepancy can be explained in two ways.

1. Theoretically, the capillary pressure head $\psi_c(z,t)$ for the dynamic case is less than for the static case of equilibrium based on $\psi_b(z_b,t)$. This fact is evident from an integrated form of Darcy's law

$$\psi_c = \psi_b - \int_z^{z_b} \left(\frac{q}{K} - 1 \right) dz \quad 0 \leq z \leq z_b \tag{17}$$

where q = positive downward flux density (LT^{-1}). Clearly, for any finite q, ψ_c is less at all $z<z_b$ than for the static case when q = 0. A smaller ψ_c in the dynamic case than in the static case requires that the water content be higher in the dynamic case. Thus, the quantity drained as predicted from equation (10) would be too large. Again, according to equation (17), the equilibrium conditions would be well approximated when the drainage rate, q, is small compared with hydraulic conductivity, K. This occurs when the drainage rate becomes small at large times.

Furthermore, equation (9) gives the rate of change of elevation of the top of the capillary fringe as directly dependent upon the drainage rate q_b. Obviously, equation (9) is a better approximation when q_b is small compared to K_s (i.e., when equilibrium conditions prevail). From Darcy's law (Eq. 12), the initial values of q_b are 0.41 K_s and 0.24 K_s for the columns of Poudre sand and Ramah sand, respectively.

2. Assuming the upper limit of the saturation zone is the top of the capillary fringe, equation (11) was written

directly from Darcy's law. Observation during the experiments showed that the upper limit of the saturation zone is below the predicted ordinate z_b after initiation of drainage (i.e., ψ_b is overestimated for each column). Strictly speaking, equation (11) was used here for a region that was not completely saturated at the upper portion. On the other hand, unsaturation causes a reduction in the cross-sectional area available for the flow and an increase in tortuosity of the flow paths. The combined effect causes a rather rapid reduction in the hydraulic conductivity which in turn causes an increase in resistance to flow. It is believed, however, that equation (11) does not account correctly for the effects of resistance to flow for the case of this study, and therefore, equation (14) causes the scaled time corresponding to a certain value of Q to be underestimated. It is worth mentioning that the theory models resistance to flow correctly at small times. This is because the column is completely saturated before initiation of the drainage.

INFLUENCE OF SOIL PARAMETERS UPON THEORETICAL CURVES AND EXPERIMENTAL DATA

The effect of θ_r, ϕ, and K_s on the equations presented before is the same as on the experimental data. This is because they appear in the scaling factors for $\hat{Q}$ and $\hat{t}$.

The effects of ψ_b and λ are not the same for both experimental and theoretical values. Changing λ affects only the theoretical values of $\hat{Q}$ and $\hat{t}$ because it enters into the equations. A change in ψ_b, however, affects both the theoretical and, via an artifact of prescribing the experimental values of $\hat{Q}$ and $\hat{t}$ because ψ_b appears in the scaling factors and enters into the equations. This effect is different than that for ϕ and θ_r which appear only in the scaling factor.

SUMMARY AND CONCLUSIONS

By using a succession of equilibrium water content distributions in the partially saturated zone during transient drainage, an approximate solution for drainage of a liquid from a vertical soil column was obtained.

Two laboratory experiments were performed to test the validity of the solution. Comparisons with experimental data show good agreement for a situation in which the assumption of a succession of equilibrium states is expected to be least adequate. Agreement between the solution and experimental data is quite good when the rate of drainage is small relative to K_s.

It is speculated that the assumption of a succession of equilibrium distributions is less adequate for one-dimensional drainage than for relief drains because of the differences in drainage rate.

NOTATIONS

The following symbols are used in this paper:

D Soil moisture diffusivity $[L^2T^{-1}]$

H_s Equivalent saturated height [L].

$\hat{H}_s$ Scaled equivalent saturated height equal to H_s/ψ_b.

K Hydraulic conductivity $[LT^{-1}]$.

K_s Saturated hydraulic conductivity $[LT^{-1}]$.

L Length of draining column [L].

$\hat{L}$ Scaled length of column equal to L/ψ_b.

q Volume flux density $[LT^{-1}]$.

q_b Volume flux at top of capillary fringe equal to drainage rate $[LT^{-1}]$.

Q Cumulative drainage flux density $[L^3L^{-2}]$.

$\hat{Q}$ Scaled cumulative drainage flux density equal to $Q/\psi_b(\phi-\theta_r)$.

t Time [T].

$\hat{t}$ Scaled time equal to $tK_s/\psi_b(\phi-\theta_r)$.

z Vertical coordinate (positive downward) [L].

z_b Coordinate of the top of the capillary fringe [L].

$\hat{z}_b$ Scaled coordinate of the top of the capillary fringe equal to z_b/ψ_b.

θ Volumetric water content.

θ_r Non-drainable water content.

ψ Soil moisture suction head [L].

ψ_b Soil moisture suction head at the top of the capillary fringe [L].

ψ_c Capillary pressure head [L].

λ Brooks-Corey pore-size distribution index.

ϕ Porosity

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