Iran Agricultural Research 9:121-138 (1990)

CRITICAL STABILITY NUMBER IN ROCK LINED CHANNELS

M. SHAFAI- BAJESTAN[1]
Department of Irrigation, Shahid Chamran University, Ahwaz, Iran.
(Received October 15, 1989)

ABSTRACT

On the basis of analysis of a particle attacked by hydrodynamic forces on the bed of an open channel, and the use of a wide range of data obtained from other sources, a relationship relating mean velocity and depth of flow to the rock properties at incipient motion is presented. Comparison of this relationship with six other methods illustrates its applicability.

تحقیقات کشاورزی ایران
۹: ۱۲۱-۱۳۸ (۱۳۶۹)

عدد پایداری بحرانی در کانالهای سنگی

محمود شفاعي بجستان
استادیار بخش آبیاری دانشگاه شهید چمران (اهواز)

چکیده

در این مقاله با در نظر گرفتن ذره سنگی در کف یك کانال که تحت تأثیر نیروهای وارده از طرف آب قرار دارد، و بکار بردن اطلاعات بدست آمده از منابع علمی مختلف رابطه ای که سرعت و عمق متوسط جریان را به خصوصیات سنگ در شرایط آستانه حرکت ربط دهد، بدست آمد. مقایسه این رابطه با روشهای متداول نشان می دهد که روش اخیر نه تنها مفید می باشد بلکه در دامنه تغییرات بیشتری از شرایط جریان می تواند مورد استفاده قرار گیرد.

1- *Assistant Professor.*

INTRODUCTION

Two approaches are often used to determine the suitability of riprap material for erosion control on the bed or sides of open channel. The first approach is based on the angle of repose of the rock material, Φ.

The riprap is considered to move if the ratio of driving force to the resisting force exceeds Tan Φ. The definition of anlge of repose, however, has been a matter of discussion among investigators. White (25), Kalinske (8), Lane (9), Lane and Carlson (19), Brooks (3), Olivier (15) and others defined the angle of repose as the angle on which particle is attacked by hydrodynamic forces, but others such as Stevens *et al.* (21), and Ulrich (23) defined it as the maximum side slope on which the rock can be stable.

Consequently a unique angle of repose cannot be used for all of these methods. Ulrich (23) recently introduced the bearing angle instead of angle of repose, but it has not been verified yet.

The second approach is to relate the incipient motion of riprap material to such flow characteristics as shear stress or mean flow velocity.

The concept of critical shear stress introduced by Shields (19) has been used in many problems of alluvial channels including riprap design. In this approach the value of the so called Shields parameter has an important role. This parameter is a function of R_*, boundary Reynolds number. At higher values of R_* it has a constant value and has been found by Shields to be about 0.06. Sequent studies, however, proposed other values ranging from 0.022 (6) to 0.062 (1). More recently Maynord (1987) argued that the Shields parameter varies with relative roughness. Simons and Senturk (20) stated that determination of shear stress involves major difficulties and the use of flow velocity is often accepted as the most important factor. Because of importance of flow velocity, many empirical relationships have been proposed to relate the incipient motion of the particle to the mean flow velocity. But because of the lack of accurate or wide range of data, a general equation applicable to a wide range of relative roughnesses has not yet been developed. It is the purpose of this paper to develop a general but simple relationship for incipient motion of the bed material based on theoretical considerations of the stability of a particle on the bed attacked by hydrodynamic forces and verify this relationship with experimental data obtained by other investigators. A comparative analysis of this method and others will be presented at the end of this paper.

THEORETICAL CONSIDERATION

The process of rock movement on the bed of an open channel can be analyzed on the basis of stability of a particle acted upon by hydrodynamic forces. These forces, which cause instability of the particles, are lift, which acts normal, and drag, which is in the flow direction. In many analyses the lift force does not appear explicitly because it has been stated that the lift depends on the same variables as drag. But studies of Einstein and El- Samni (5), Urbonas (24) and others showed that the lift is very important in entraining sediment especially when a particle is sheltered by other particles. Drag force also is important because it causes particles to move. Thus, the combination of lift and drag forces from moving fluid causes the displacement of particles. Referring to Fig. 1 the driving force, f_{dr} , can be written as:

$$F_{dr} = (F_L^2 + F_d^2)^{0.5} = F_L [1 + (F_d/F_L)^2]^{0.5} \qquad (1)$$

Where F_L = The lift force and F_d = drag force.

Samad (17) used the existing data and developed the ratio of lift to drag force (F_L/F_d) as a function of boundary Reynolds number ($R_* = u_* D_s/\nu$ where u_*= shear velocity, and ν= kinematic viscosity). According to this study when R_* is greater than 3000, the ratio of lift to drag is constant. Thus, assuming a relatively large boundary Reynolds number, Eq (1) can be written as:

$$F_{dr} = C_l F_L \qquad (2)$$

Where $C_l = [1+(F_D/F_l)^2]^{0.5}$= constant.

F_L or the lift force is due to differences in pressure on the top surface and the bottom of a particle on the channel bed. This difference in pressure is due to the velocity fluctuation in the vicinity of the particle. The lift force to which the particle is subjected is not constant but fluctuates with time because of the turbulent velocity field adjacent to the particle (5,8).

The lift force is expressed as:

$$F_L = C_L \rho A \, V_r^2/2 \qquad (3)$$

Where C_L is the lift coefficient which depends on Reynolds number but For relative large R_e, C_L can be assumed a constant, A= projected area and V_r= flow velocity in the vicinity of the particle.

Einstein (4) in developing of his bed load equation proposed a value of 0.178 for C_L.

The projected area, A, can be written as a function of rock size in the form $C_2D_s^2$, where C_2 depends on the particle shape. For sphere the value for C_2 is $\pi/4$.

Substituting Eq.3 in Eq.2, the driving force can be written as:

$$F_{dr} = C_3 \rho D_s^2 V_r^2 \quad (4)$$

in which $C_3 = C_1C_2$.

Referring to Fig. 1, the resisting force which causes particle to be stable on the bed of horizontal channel, is the combination of buoyancy force, F_B, acting normal to the water surface and the weight of particle, W, acting normal to the bottom, thus:

$$F_R = W - F_B = C_4\gamma_S D_S^3 - C_4\gamma\omega D_S^3 = C_4(\gamma_S - \gamma\omega)D_S^3 \quad (5)$$

Where γ_S and $\gamma\omega$ are the unit weight of particle and water, respectively, and, C_4= constant depends on the particle shape, for sphere $C_4 = \pi/6$.

Combining Eqs (4) and (5), the ratio of driving force to resisting force is:

$$R = C_5\ \rho V_r^2 / (\gamma_S - \gamma)D_S = C_5\ V_r^2 / g(S_S - 1)D_S \quad (6)$$

Where C_5= coefficient depends on boundary Reynolds number and the Aarticle shape but for large R_e and given particle it is constant and, S_s = specific weight of the bed material.

When R, the ratio of driving force to resisting force is greater than one, the particles are in motion and when R is smaller than one the particles are expected to be stable. For a given particle shape and

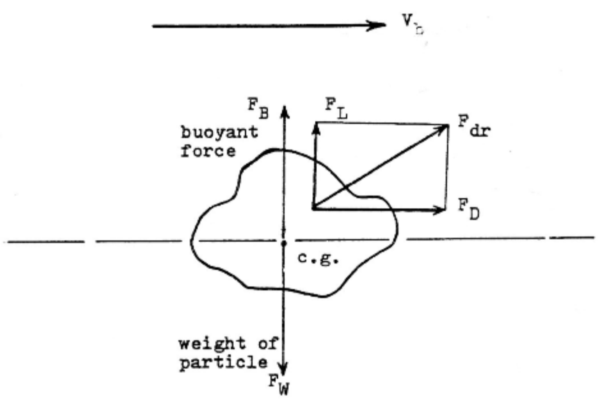

Fig. 1. Definition of forces acting on a particle.

relatively large boundary Reynolds number, the incipient motion can be expressed as:

$$V_r^2 / g(S_S-1)D_S = C_6 \tag{7}$$

In which C_6 is constant. Since the measurment of V_r is difficult, it is appropriate to relate V_r to the mean flow velocity and relative roughness by means of log-law velocity distribution. The general form of such relationship at hydraulically rough boundary surfaces is:

$$V_r = f(V, d/D_S) \tag{8}$$

in which d is the average flow depth. Combining Eq.8 with Eq.7 gives:

$$f\ (V^2/g(S_S-1)D_S\ ,\ d/D_S) = 0 \tag{9}$$

The square root of $V^2/g(S_S-1)D_S$ will be called "stability number" and is shown as "SN". By this definition Eq. 9 can be written as:

$$SN = f\ (d/D_S) \tag{10}$$

or,

$$SN = a\ (d/D_S)^n \tag{11}$$

Eq. 11 is a general functional relationship relating the stability of a particle to the flow conditions at the point of incipient motion for a particle on the bed of an open channel. To determine values of a and n, experimental date from other sources were taken as follows:

1) CSU (1986)'s Data

The first series of data (data No. 1 through No. 35 in Table 1) are obtained from Abt *et al.* (1). This study was conducted at Colorado State University. The purpose of this study was to provide the riprap design criteria for overtopping flows. Two flumes; outdoor (3.66 m or 12 ft wide) for simulating steep slopes greater than 10% and indoor flume (2.44 m or 8 ft wide) for simulating flatter slopes were used. Five different sizes of riprap material ranging from 2.24 cm (1 in.) to 15.24 cm (6 in.) were tested.

Table 1. Data used in this study.

Data No. (1)	Run No. (2)	D_{50} ft (3)	S_s (4)	S (5)	d ft (6)	V fps (7)	D_{50}/d (8)	SN (9)	Remarks (10)
1	6o	0.342	2.65	0.2	0.20	5.59	1.71	1.31	Failure
2	7-o	0.342	2.65	0.2	0.22	6.62	1.55	1.56	Failure
3	9-0	0.425	2.65	0.2	0.26	6.53	1.64	1.40	No Failure
4	13-0	0.517	2.65	0.2	0.35	5.92	1.48	1.13	No Failure
5	17-0	0.183	2.72	0.2	0.10	3.45	1.83	1.08	Failure
6	18-0	0.183	2.72	0.2	0.11	3.49	1.66	1.10	Failure
7	1-I	0.183	2.72	0.02	0.57	5.34	0.32	1.69	No Failure
8	2-I	0.183	2.72	0.02	0.84	7.32	0.22	2.30	Failure
9	3-I	0.183	2.72	0.02	0.81	6.77	0.23	2.12	No Failure
10	4-I	0.183	2.72	0.02	0.85	6.87	0.22	2.16	No Failure
11	5-I	0.183	2.72	0.02	0.93	7.08	0.20	2.22	No Failure
12	6-I	0.183	2.72	0.02	1.00	7.07	0.18	2.22	Failure
13	7-I	0.183	2.72	0.02	0.74	6.12	0.25	1.92	Failure
14	8-I	0.085	2.72	0.02	0.32	3.91	0.27	1.80	No Failure
15	9-I	0.085	2.72	0.02	0.41	4.06	0.21	2.12	No Failure
16	10-I	0.085	2.72	0.02	0.47	4.87	0.18	2.24	Failure
17	11-I	0.085	2.72	0.02	0.30	3.65	0.28	1.68	No Failure
18	12-I	0.085	2.72	0.02	0.31	3.44	0.27	1.59	No Failure
19	13-I	0.085	2.72	0.02	0.38	3.91	0.22	1.80	No Failure
20	14-I	0.085	2.72	0.02	0.43	4.35	0.20	2.00	No Failure
21	15-I	0.085	2.72	0.02	0.42	5.30	0.20	2.44	Failure
22	16-I	0.085	2.72	0.02	0.34	4.41	0.25	2.03	No Failure
23	17-I	0.085	2.72	0.02	0.40	4.74	0.21	2.18	No Failure
24	18-I	0.085	2.72	0.02	0.42	5.37	0.20	2.47	Failure
25	19-I	0.085	2.72	0.01	0.41	3.07	0.21	1.41	Failure
26	21-I	0.085	2.72	0.01	0.80	4.95	0.11	2.28	Failure
27	22-I	0.085	2.72	0.01	0.86	5.03	0.10	2.32	Failure
28	23-I	0.085	2.72	0.01	0.94	5.40	0.09	2.49	Failure
29	24-I	0.085	2.72	0.01	1.00	5.38	0.09	2.48	Failure
30	26-I	0.085	2.72	0.10	0.08	4.10	1.06	1.89	Failure
31	27-I	0.085	2.65	0.10	0.11	2.84	0.77	1.31	Failure
32	28-I	0.085	2.65	0.10	0.14	2.98	0.61	1.37	Failure
33	29-I	0.183	2.65	0.10	0.23	5.15	0.80	1.62	Failure
34	30-I	0.183	2.65	0.10	0.25	5.00	0.73	1.57	Failure
	Second series of data. Ruff et al. (1985)								
35	6	0.083	2.68	0.01	0.69	4.6	0.12	2.2	Failure
36	7	0.083	2.68	0.009	0.71	4.8	0.12	2.3	Failure
37	22	0.083	2.68	0.003	1.8	4.6	0.05	2.4	Failure
38	26	0.083	2.68	0.003	2.3	5.15	0.04	2.4	Failure
39	36	0.083	2.68	0.013	0.62	4.95	0.13	2.3	Failure
40	32	0.083	2.68	0.006	1.3	4.9	0.06	2.3	Failure
41	41	0.083	2.68	0.005	1.35	4.7	0.06	2.2	Incipient
42	39	0.083	2.68	0.004	1.84	5.0	0.05	2.4	Failure
43	40	0.083	2.68	0.003	1.92	4.8	0.04	2.2	Failure
44	44	0.083	2.68	0.004	2.21	5.2	0.04	2.5	Failure
45	78	0.167	2.64	0.016	1.02	6.14	0.16	2.2	Incipient
46	69	0.167	2.64	0.019	0.95	6.6	0.18	2.2	Incipient
47	71	0.167	2.64	0.008	1.48	6.3	0.12	2.2	Failure
48	75	0.167	2.64	0.010	1.80	7.0	0.09	2.4	Failure
49	84	0.167	2.64	0.019	0.97	6.6	0.17	2.2	Failure
50	88	0.167	2.64	0.016	1.4	6.8	0.12	2.3	Failure
51	94	0.167	2.64	0.013	1.6	8.02	0.09	2.7	Failure
52	27	0.083	2.68	0.003	2.3	5.1	0.04	2.4	Incipient
53	37	0.083	2.68	0.012	0.65	4.8	0.13	2.3	Incipient
54	45	0.083	2.68	0.004	2.33	5.1	0.04	2.4	Incipient
55	67	0.167	2.64	0.015	1.0	6.4	0.17	2.2	Incipient
	Third series of data. Maynord (1978)								
56		0.026	2.68	0.008	0.85	2.8	0.03	2.36	
57		0.026	2.68	0.008	1.00	2.78	0.026	2.36	
58		0.026	2.68	0.068	1.13	2.79	0.023	2.36	

1 ft = 0.3048 m

1 fps = 0.3048 m Sec^{-1}

Table 1. Continued

Data No. (1)	Run No. (2)	D_{50} ft (3)	S_s (4)	S (5)	d ft (6)	V fps (7)	D_{50}/d (8)	SN (9)	Remarks (10)
59		0.026	2.68	0.008	1.24	2.83	0.021	2.39	
60		0.032	2.68	0.008	0.81	3.0	0.04	2.28	
61		0.032	2.68	0.008	0.96	2.95	0.033	2.24	
62		0.032	2.68	0.008	1.08	2.98	0.03	2.26	
63		0.032	2.68	0.008	1.19	3.01	0.027	2.29	
64		0.037	2.68	0.008	0.79	3.1	0.047	2.2	
65		0.037	2.68	0.008	0.91	3.18	0.041	2.25	
66		0.037	2.68	0.008	1.04	3.15	0.036	2.2	
67		0.037	2.68	0.008	1.17	3.1	0.032	2.2	
68		0.026	2.68	0.008	0.92	2.8	0.028	2.36	
69		0.026	2.68	0.008	1.08	2.8	0.024	2.37	
70		0.026	2.68	0.008	1.22	2.84	0.021	2.39	
71		0.032	2.68	0.008	0.86	3.1	0.03	2.33	
72		0.032	2.68	0.008	1.01	3.08	0.026	2.34	
73		0.032	2.68	0.008	1.18	2.98	0.022	2.26	

1 ft = 0.3048 m

1 fps = 0.3048 m Sec^{-1}

For each run by the use of accurate instrumentation, the water surface elevation and flow velocity through and over the riprap layer were measured.

2) CSU (1985)'s Data

These series of data (No. 35 through No. 55 in Table 1) were obtained from a study by Ruff *et al.* (16) which was conducted at Colorado State University. The purpose of this study was to provide the design criteria of rock riprap as a flood control. Here, the 8-ft flume (2.44 m wide) was used. Two sizes, 2.54 cm (1 in.) and 5.08 cm (2 in.), of crushed limestone with the same gradation were tested. For each run, the water surface elevation and the velocity profile were recorded.

3) The Third Series of Data, Obtained from Maynord (12)

Three rock sizes ranged from 0.74 cm (0.026 ft) to 1.13 cm (0.037 ft) were tested on trapezoidal cross section having bottom width of 152.4 cm (5ft) and various side slopes. In each experiment, a constant discharge was used.

The tailwater was then lowered in small increments until failure of the riprap material occurred. Failure was assumed to represent the point at which the rock began to move. Three types of failure were reported: on the channel bed only, channel bed and side slope, and channel side slope only. In this study the data related to the first two types of failure will be used.

ANALYSIS OF DATA

The stability number and relative roughness for all data, described in the previous section, were computed and plotted on log-log paper as shown in Fig. 2. A conservative line plotted so that; *a)* it passes through most of the point corresponding the initial movement of the rock, and *b)* the failure

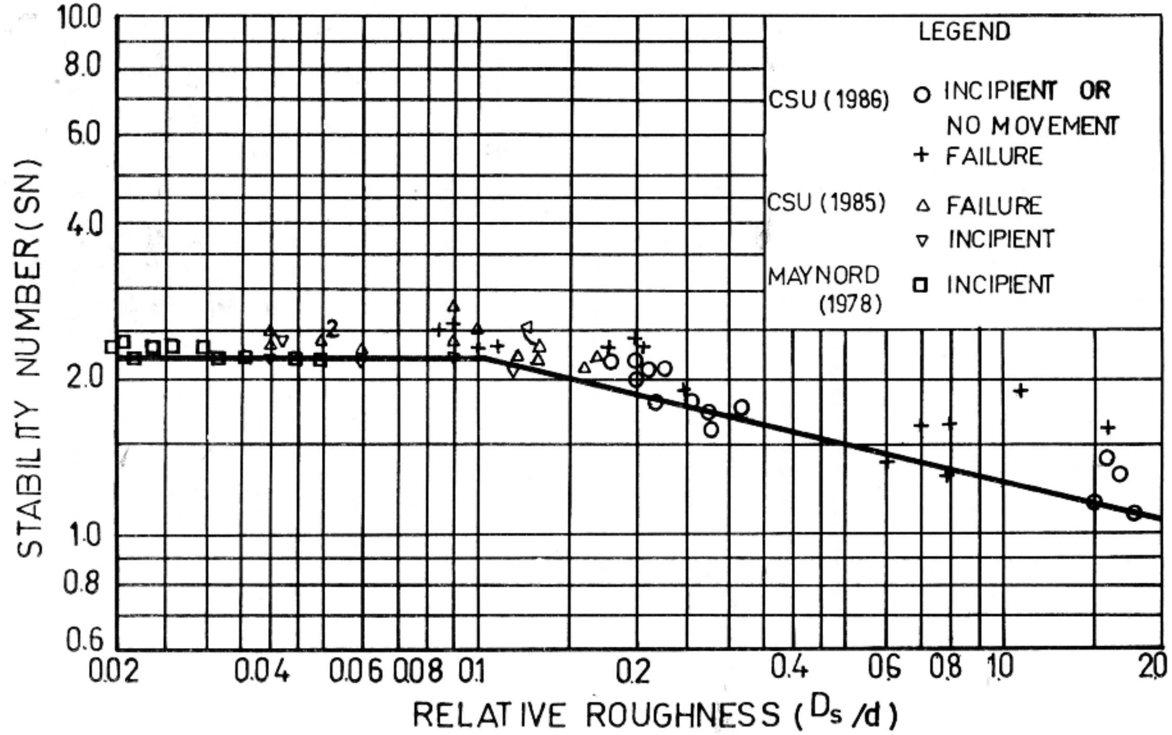

Fig. 2. Variation of stability number with relative roughness, experimental data and the proposed method.

points to be located above this line. Consequently, this line represents the criterion of incipient motion of the rock on the bed of an open channel. Mathematically this line represents the following equations:

$$SN = 2.2 \qquad \text{For } D_{50}/d < 0.1 \qquad (12)$$

$$SN = 1.237\,(d/D_{50})^{0.25} \qquad \text{For } D_{50}/d > 0.1 \qquad (13)$$

It was noted that the stability number is constant for small relative roughness. For larger relative roughness, as was expected, the particle is less stable as the ratio of D_{50} to d increases.

To compare the new relationships with several of the more widely recognized existing formulas, Fig. 3 was developed. The design methods used for this comparison are those by Isbach (7), Straub (22), Neill(13), Neill (14), Bogardi (2) and Maynord (12) [for complete review of these studies see Shafai-Bajestan (18)]. Table 2 Shows the original form of equations proposed by these investigators as well as the coefficients a and n if these formulas are rewritten in the form of Eq (11).

The comparison shows a general agreement between the new relationship and other studies. However, each of these methods is applicable only for a limited range of relative roughness. For example, when $D_{50}/d > 0.3$, almost all methods underestimate the required size of riprap.

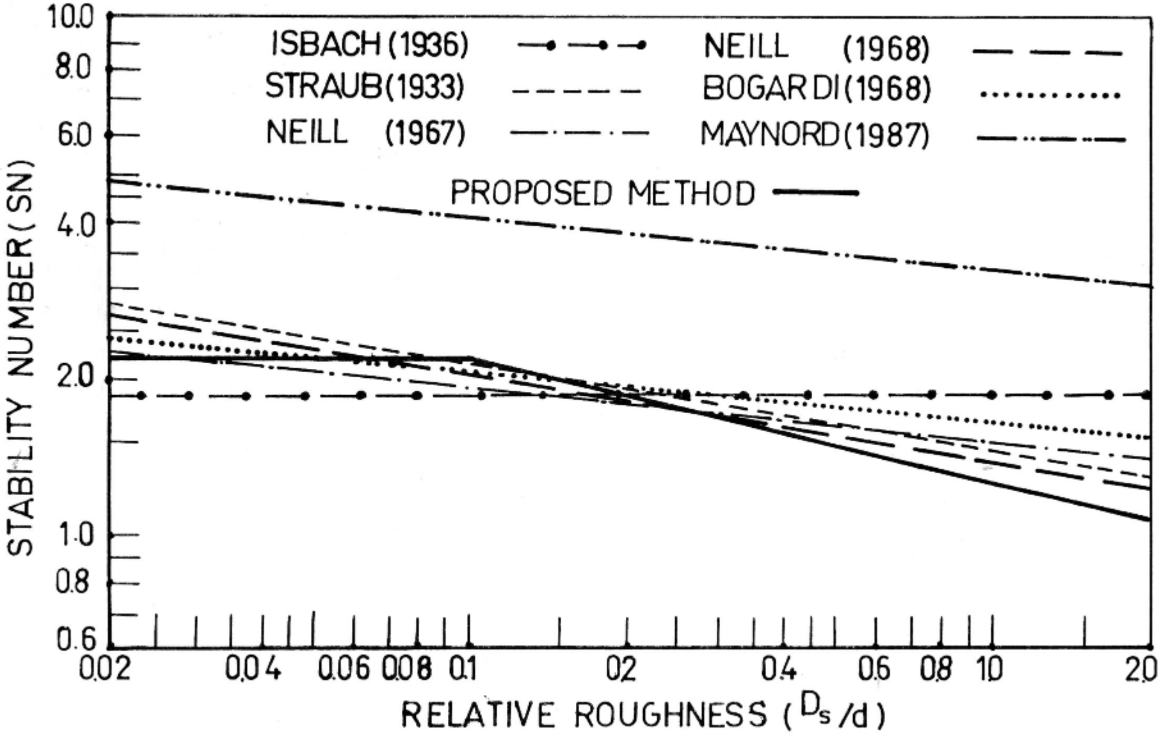

Fig. 3. Comparison of the proposed method with other existing methods.

Table 2. Summary of existing formulas

Investigator	Original form of equation	a	n
Isbach(1936)	$V_c = 1.2\,[\,2g(\frac{\rho_s - \rho}{\rho})D_{50}\,]^{0.5}$	1.7	0.0
Straub(1953)	$V_c = 8.45\,(\frac{\rho_s - \rho}{\rho})^{0.5}\,(\frac{d}{D_s})^{1/6}\,D_s^{0.5}$	1.49	1/6
Neill(1967)	$\frac{\rho V_c^2}{g(\rho_s - \rho)D_{50}} = 2.5\,(\frac{D_{50}}{d})^{-0.20}$	1.58	0.10
Neill(1968)	$\frac{\rho V_c^2}{g(\rho_s - \rho)D_{50}} = 2.0\,(\frac{D_{50}}{d})^{-1/3}$	1.41	1/6
Bogardi(1968)	$\frac{V_c}{[g(S_s - 1)d\,]^{0.5}} = 1.7\,(\frac{d}{D_s})^{-0.405}$	1.70	0.095
Maynord(1987)	$\frac{V_c}{[g(S_s - 1)d\,]^{0.5}} = 3.33\,(\frac{d}{D_s})^{-0.20}$	3.33	0.10

SUMMARY AND CONCLUSION

This paper discusses the results of an investigation to develop a relationship for incipient motion of sediment particles on the bed of an open channel. Such a relationship is useful in many practical problems such as the design of stable channel, stabilizing the bed of an open channel or protecting the embankment from overtoping flow.

On the basis of stability analysis of a particle on which hydrodynamic forces act, a general relationship at the point of incipient motion was developed. This relationship was checked using wide range of data obtained from three sources. This relationship relates the mean flow velocity to the particle properties at the threshold conditions.

Comparison of the proposed equations with six widely recognized existing formulas illustrate the applicability of this method and shows that those methods are applicable for limited range of relative roughnesses.

ACKNOWLEDGEMENT

The author wishes to express his highest gratitude to Professor M.L. Albertson of Colorado State University, who introduced the author to the fascinating field of sediment transport. I would like to thank the reviewers, Professor R.W. Jeppson and two anonymous persons, for their thoughtful reviews.

LITERATURE CITED

1. Abt. S.R., Ruff, J.F., Shaikh, A. and E.V. Richardson. 1986. Environmental assessment of uranium recovery activities-Riprap testing phase I. Dept. of Civil Eng., Report No. CER 85/86/24, CSU, Ft Collins.
2. Bogardi, J.L. 1968. Incipient sediment motion in terms of critical mean velocity. Acta Technical Academiae Scientarium Hungaricae, Budapest 62:1-24.
3. Brooks, N.H. 1963. Discussion of boundary shear stresses in curved trapezoidal channels, by Ippen and Drinker, J. Hydraulics Division, ASCE, 89:327-333.
4. Einstein, H.A. 1950. The bed-load function for sediment transportation in open channel flows. Technical Bulletin No.10 26, U.S. Department of Agriculture.
5. Einstein, H.A. and El-Samni, E.S.A. 1949. Hydrodynamic forces on rough wall. Review of Modern Physics, 21:520-524.
6. Feton, J.D. and Abbott, J.E. 1977. Initiad movement of grains on streambed: The effect of relative protrusion. Proc. Royal Soc. London, Vol. 353A: 1091-1095.
7. Isbash, S.V. 1936. Construction of dams by depositing rock in running water. 2nd Congress on Large Dams, Paper C2:125-136.
8. Kalinske, A.A. 1947. Movement of sediment as bed load in rivers. Transaction, American Geophysical Union, 28:615-620.
9. Lane, E.W. 1955. Design of stable channels. Transaction, ASCE, 120:1234-1279.
10. Lane, E.W. and Carlson, E.J. 1953. Some factors affecting the stability of canal constructed in coarse granular materials. Proceeding of 5th congress of IAHR, Minnesota, International Hydraulic Convention. Minneapolis, U.S.A.

11. Maynord, S.T. 1978. Practical riprap design. Miss. Paper H-78-4, U.S. Army Engineering Waterways Experiment Station, Vicksburt, Miss., U.S.A.

12. Maynord, S.T. 1987. Stable riprap for open channel flows. Ph.D. Dissertation, Dept. of Civil Eng., CSU, Ft Collins, Colorado. U.S.A.

13. Neill, C.R. 1967. Mean velocity criterion for scour of coarse uniform bed material. Proceeding of 12th Congress of IAHR, paper C6, Ft Collins, Colorado, U.S.A. 46-54.

14. Neill, C.R. 1968. Note on initial movement of coarse uniform bed material. Hydraulic Res. IAHR, 6:173-176.

15. Olivier, H. 1967. Through and overflow rockfill dams-new design techniques. Proc. Ins. of Civil Eng., 36:433-471.

16. Ruff, J.F. Shaikh, A., Abt, S.R. and Richardson, E.V. 1985. Riprap tests in flood control channels. Report No. CER. 85-86-JFR-AS-SRA-EVR-17, Dept. of Civil Eng., CSU, Ft Collins, Colorado, USA.

17. Samad, A. 1978. Analysis of riprap for channel stabilization. Ph.D. Dissertation, Dept. Civil Eng., CSU, Ft Collins, Colorado, USA.

18. Shafai-Bajestan, M. 1989. Riprap criteria at pipe outlet. Ph. D. Dissertation, Dept. Civil Eng., CSU, Ft Collins, Colorado, USA.

19. Shields, A. 1936. Application of similarity principles and turbulence research to bed-load movement. English translation by W.P. Ott and J.C. Van Uchelon, California Inst. Technol. Publication 167.

20. Simons, D.B. and Senturk. 1977. Sediment Transport Technology. Water Resource Publication, Ft Collins, Colorado. U.S.A.

21. Stevens, M.A., Simons, D.B. and Lewis, G.L. 1946. Safety factors for riprap protection. Hydraulic Eng., ASCE, 10:634-657.

22. Straub, L.G. 1953. Dredge fill closure of Missouri river at Fort Randall. Proc. Minnesota International Hydraulics Convention, Minneapolis, Minnesota, U.S.A. 61-35.

23. Ulrich, T. 1987. Stability of rock protection on slopes. J. Hydraulic Eng., ASCE, 113:849-891.

24. Urbonas, B.R. 1968. Forces on a bed particle in a dumped rock stilling basin. M.S. Thesis, Dept. Civil Eng., CSU., Ft Collins, Colorado, USA.

25. White, C.M. 1940. The equilibrium of grains on the bed of a stream. Proc. of Royal Soc. 174: 322 - 338.

Symbols used in this paper

A = projected area
a = a dummy variables
D_{50} = median particle size
D_s = representative of riprap material size
d = flow depth
F_B = buoyancy force
F_D = drag force
F_{dr} = driving force
F_L = life force
F_R = resisting force
g = acceleration of gravity
n = a dummy variable
R = ratio of driving force to resisting force
S_s = specific gravity
u_* = shear velocity
V = average velocity
V_r = bottom velocity
W = force due to weight of particle
ρ = mass density of water
γ = kinematic viscosity
γ_s = unit weight of particle
$\gamma\omega$ = unit weight of water
π = 3.14