Iran Agricultural Research 15:1-18 (1996)

DECISION CRITERIA IN RISK ANALYSIS: AN APPLICATION OF STOCHASTIC DOMINANCE WITH RESPECT TO A FUNCTION

J. TORKAMANI[1]

Department of Agricultural Economics, College of Agriculture, Shiraz University, Shiraz, Iran.

(Received: May 13, 1995)

ABSTRACT

Decision criteria under risk are briefly discussed and evaluated. It is concluded that stochastic dominance with respect to a function is more flexible and powerful than other decision rules. By placing limits on the decision maker's degree of absolute risk aversion, this method reduces a large number of actions to an efficient set that are not dominated for the specified class of decision makers. Thus, there is no need for detailed knowledge of the decision maker's preferences. Using utility efficient programming model, it was illustrated that this procedure can effectively incorporate the stochastic

1. Assistant Professor.

dominance with respect to a function criterion in whole-farm planning. Results of this study demonstrated that risk aversion plays an important role in farmers' behavior. Ignoring this factor can lead to selecting programs which overestimate farmers' levels of total net revenue. This may explain why results of deterministic models based on assumed risk neutrality are often disregarded by farmers as unrealistic.

تحقیقات کشاورزی ایران

۱۵:۱-۱۸ (۱۳۷۵)

معیارهای تصمیم‌گیری در شرایط عدم اطمینان: کاربرد روش "برتری استوکستیک با توجه به فرم یک تابع"

جواد ترکمانی

استادیار بخش اقتصادکشاورزی دانشکده کشاورزی دانشگاه شیراز، شیراز، ایران.

چکیده

معیارهای مختلف تصمیم‌گیری در شرایط عدم اطمینان وخطر مورد بررسی و ارزیابی قرارگرفت است. پس از، اشاره به مزایای روش "برتری استوکستیک با توجه به فرم یک تابع"، کاربرد آن در چهارچوب روش "برنامه‌ریزی راندمان مطلوبیت" به نمایش گذاشته شده است. با توجه به مشکلات و مسائل احتمالی تخمین تابع مطلوبیت، روش فوق می تواند با استفاده از

حدهای تحتانی و فوقانی ضریب خطرگریزی دامنه برنامه‌های بهینه را محدود کند و نقش مؤثـری در تعیین برنامه بهینه بهره‌برداری های کشاورزی در شـرایط عـدم اطمینـان و خطـر داشـته باشـد. نتـایج مطالعه موردی نشان داد که عدم اطمینان و طبیعت بهره بـرداران در برخـورد بـا آن نقـش مهمـی در تصمیم‌گیری های کشاورزان دارد. شاید این یکی از دلایل عمده عدم توجه بهره‌بـرداران کشـاورزی به تعدادی از برنامه‌هائـی باشد که با استفاده از روش های بیشینه کننده سـود و بـا فـرض بی‌تفـاوتی بهره‌برداران در رویارویی با مخاطرات تهیه شده‌اند.

INTRODUCTION

Decisions under imperfect knowledge are usually considered to be risky. Dillon and Hardaker (11) define risk as "a situation with uncertain consequences". Rae (28) argued that farmers always face non-certainty due to lack of perfect knowledge about the results of their decisions. In agricultural production, risk may arise as a result of unpredictable yields, product prices, and production costs (2, 21). Yield uncertainty is mainly due to factors such as extremes of climate (e.g., temperature and rainfall), pests and disease problems (15). A major source of risk is price uncertainty. Farmers do not have control over the market in which planned production will be sold. Price variations arise mainly from changes in aggregate demand and supply which are beyond the farmer's control. Under these circumstances the farmer's decision may be regarded as risky.

The objectives of this study were to (i) examine and evaluate the decision criteria under risk and uncertainty, (ii) demonstrate an application of the technique of stochastic dominance with respect to a function (SDWRF) to a farm decision problem, and (iii) illustrate the importance of risk attitude in farmers' behavior. The guiding hypothesis is that farmers' decision may be regarded as risky and hence it seems rational to incorporate their attitude toward risk in the production analyses.

In the following parts, a brief evaluation of Bernoullian decision theory, efficiency criteria and safety-first rules criteria as well as the report of results from application of SDWRF are presented.

Bernoullian Decision Theory

The Bernoullian principle provides a unique way of ranking risky prospects. It combines the degrees of belief of the decision- maker regarding uncertain events with their degree of preference for particular consequences in a formal theory of choice (2, 29).

Despite its wide acceptance, the subjective expected utility model (SEUM) has been criticized as inadequate for describing real decision making situations (12, 30, 32). Hence, alternative models have been proposed.

Safety-First Rules

In these decision rules, risk is defined as the probability that some goal such as expected return assumes a value below a specific disaster level. Safety-first rules, in general, imply a preference for survival of the firm before the profit-oriented objectives (7, 31, 33).

Safety-first models suffer from several drawbacks. They need the specification of an arbitrary exogenous disaster level. Furthermore, unlike SEUM, they lack a sound axiomatic foundation and, therefore, do not provide a mechanism to study decision making in a complex situation when multiple attributes and uncertainties are involved.

Efficiency Criteria

The expected utility model has provided the most accepted mechanism for ranking risky prospects in order of preference, the most preferred prospect being the one with highest expected utility. However, practical difficulties may be associated with empirical estimation of the decision maker's utility function. Efficiency criteria have been devised to assist the choice without specifying the utility function except to a limited extent.

They allow a partial ordering of risky prospects for decision makers based on reasonable assumptions about the nature of their utility functions. Thus, there is no need for detailed knowledge of the decision- maker's preferences.

By placing bounds on the level of risk aversion, an efficiency criterion divides various actions into an efficient set and an inefficient set. The efficient set contains those actions that are not dominated for the specified class of decision makers (1, 2, 22).

Stochastic dominance methods are useful in comparisons of risky prospects when utility functions are not fully specified. If f(x) and g(x) are two probability density functions for a risky prospect x with cumulative distribution of R and defined over [a, b], then cumulative density functions (CDFs) of these two alternatives are as:

$$F_1(R) = \int_a^b f(x)dx, \qquad G_1(R) = \int_a^b g(x)dx$$

The stochastic dominance methods involve the pairwise comparison of these CDFs (1, 2, 22). In the following sections some of these methods are outlined.

First-degree stochastic dominance. In first-degree stochastic dominance (FSD) the only restriction on the utility function is on the first derivative. It is assumed that the utility function is monotonically increasing with the first derivative strictly positive (i.e., decision makers prefer more to less). Then, given two alternatives f and g, f is preferred to g in terms of FSD if $F_1(R) \leq G_1(R)$ for all R in the range [a, b] with at least one strong inequality. In graphical terms, this means that a FSD dominant CDF lies nowhere to the left of a dominated distribution.

The assumption of positive marginal utility implies no restrictions on risk aversion, commonly measured by the coefficient of absolute risk aversion, i.e., $r_A = - U''(x)/U'(x)$ (5, 27). The range is, therefore, from $r_1(x) = -\infty$ to $r_2(x) = +\infty$ for all non-negative values of x (22), where $r_1(x)$ and $r_2(x)$ are respectively the upper and lower bounds on the coefficient of absolute risk aversion, and $U'(x)$ and $U''(x)$ are the first and second derivatives of a Bernoullian utility function.

Because FSD has the mildest possible restriction on utility function, its discriminatory power is very limited. It is not able to 'compact' the efficient set effectively. Thus, second-degree stochastic dominance, which is a more restrictive concept of efficiency, has been introduced.

Second-degree stochastic dominance. Second-degree stochastic dominance (SSD) holds for all decision makers who are averse to risk. This implies an additional restriction on utility function compared to FSD. It is assumed that utility function over the range of [a, b] is not only monotonically increasing (i.e., $U'(x) > 0$) but also is strictly concave (i.e., the second derivative, $U''(x)$, is negative). These assumptions imply that the range of coefficient of absolute risk-aversion is from $r_1(x) = 0$ to $r_2(x) = +\infty$.

Although SSD has greater discriminatory power than FSD, it may still leave several options in the efficient set (1, 22). Thus, other efficiency criteria such as third-degree stochastic dominance and stochastic dominance with respect to a function have been developed.

Third-degree stochastic dominance. Third-degree stochastic dominance (TSD), according to Whitmore (34) and Hammond (18), is the logical extension of SSD. It holds for all individuals who become less risk averse as their wealth increases, i.e., $U'''(x) > 0$.

Under TSD, distribution f is preferred to distribution g if $F_3(R) \leq G_3(R)$ for all R over the range of [a, b] with at least one strict inequality and if $F_2(b) \leq G_2(b)$. While for TSD there are more restrictions on utility function, its discriminatory power, over SSD, in reducing the number of alternatives in the efficient set has not been proved to be significant. Hence, research to develop more flexible and powerful criteria such as stochastic dominance with respect to a function (24, 25), and extensions of stochastic dominance (8, 13) has been continued. Of these, the most widely used has been stochastic dominance with respect to a function (e.g., 3, 10, 16, 20, 23).

Stochastic dominance with respect to a function (SDWRF). The basic idea in SDWRF is to set limits on the degree of absolute risk aversion coefficient within which the group of decision makers is defined. As mentioned above, bounds already exist in the case of other efficiency criteria. Using the absolute risk aversion coefficient SDWRF, then, reduces the extent of bounds to :

$$r_1(x) \leq - U''(x)/U'(x) \leq r_2(x)$$

The coefficient of absolute aversion measures the curvature of the utility function and is a unique global measure of preference which, being a pure number, allows interpersonal comparisons of the degree of risk aversion. Bounds on this coefficient thus represent an interval of preferences (22).

SDWRF is, in fact, an extension of second-degree stochastic dominance. It is a strong ordering rule which may compact further the stochastically efficient set compared to other efficiency criteria. While it operates within the framework of SEUM, nevertheless, it avoids practical problems associated with elicitation of a single-valued utility function (4, 9, 22, 25).

Although techniques of efficiency analysis have been developed, procedures for their effective use in whole-farm planning have been limited (17, 35). Nevertheless, Patten *et al.* (26) and Hardaker *et al.* (19) have developed an approach to combining stochastic dominance criteria and whole-farm planning, i.e., utility efficient programming (UEP), which is less restrictive than the above methods. It can generate an efficient set of farm plans for farmers whose absolute risk aversion functions are defined over a specified interval. This procedure develops an efficient set very similar to that identified using the Meyer criterion (19).

METHODOLOGY

Considering the merits of UEP, SDWRF was used to determine farmers' optimal plan. The model can be simplified as:

maximize $E(U) = p'u(z)$

subject to:

$$Ax \leq b$$

$$Cx - Iz = uf$$

and $$x \geq 0$$

Note that U(.) is a monotonic and strictly concave utility function. Further, it is assumed that the utility function has a negative exponential parametric form of:

$$U = \exp[-\{(1-\lambda)a + \lambda b\}Z], \qquad \lambda \text{ parametric}$$

Variation in λ may be interpreted as variation in risk preferences, when r_A varies between a and b. λ varies from zero to one. Thus r_A is equal to a when λ is zero and close to b when λ is one.

Where z is a vector of net incomes; u (z) is a vector of utility of net revenue by state; A is a matrix of technical coefficients; p is a vector of state probabilities; C is a matrix of activity net revenue; I is an identity matrix; u is a vector of ones; x is a vector of activity levels; f is fixed costs; and b is a vector of resource stocks.

Data

Data on items such as input-output coefficients, access to farm inputs and credit, as well as resource base information were collected from a sample of 45 farmers who were selected by a two- stage cluster sampling from Kavar district near Shiraz, Fars province. First, a cluster of three villages was selected. Second, 15 farmers were chosen randomly from each village. Data were then collected using questionnaires which were designed according to the FAO farm management data collection and analysis scheme (14). The median farm was chosen as being representative after ranking the farms on the basis of area. A summary of the basic characteristics of the sample and representative farm (Table 1) allows comparison of the extent to which the selected farm was representative of total farms.

The Equally Likely Certainty Equivalent (ELCE) method with imaginary payoffs (2) was used in the present study to elicit the utility functions of sample farmers. The r_A values ranged from 0.000002 to 0.00015, hence all the sample farmers are classified as risk averse. This empirically determined range was used in the programming model to specify the range of risk aversion for the representative farm.

The model comprises the objective function, activities and constraints. The general objective of the programming models was maximization of the expected utility of the farmer's total net revenue subject to satisfying the minimum required family food needs. As mentioned above, it was assumed that utility function has a negative exponential functional form. The activities were grouped into production, selling, borrowing, purchasing and consumption activities. The objective function was maximized subject to several constraints such as land, labor availability in different periods, working capital, water availability, borrowing limit and to subsistence requirement. The UEP model of the representative farm was solved by using the General Algebraic Modeling System (GAMS/MINOS) non-linear maximization option (6)

RESULTS AND DISCUSSION

Table 1 shows that the selected farm is reasonably representative of sample farms, applying a two-standard deviation limit to the mean characteristics of the representative farm. The mean age of sample farmers was 41.25 with a standard deviation of 14.50 years. Of the total sample, 28 and 17 farmers were above 40 and 50 years, respectively. Household size was defined by the number of family member living on the farm. Table 1 indicates that family size was around seven persons for both representative farmer and total sample

Table 1. Socio- economic characteristics of representative farm and sample farms†.

Attribute	Representative farm	Sample farm
Age of farmer		
Mean (years)	43.00	41.25
SD§	n.a.	14.50
Household size		
Mean (number)	7.00	7.50
SD§	n.a.	3.24
Farm size		
Mean (ha)	6.50	5.50
SD§	n.a.	3.45

† The representative farm has 6.5 ha of operated land.

§ SD stands for standard deviation.

n.a. Denotes not applicable.

farmers. Farm size was defined by the area of land cultivated by the farmer. Mean size of sample farm was 5.50 ha which is smaller than the size of representative farm.

Table 2 illustrates the discriminatory power of SDWRF. By placing limits on the decision maker's degree of absolute risk aversion, SDWRF reduces a large number of actions to an efficient set that are not dominated for the specified class of decision makers. Thus, there is no need for detailed knowledge of the decision maker's preferences.

Table 2. Economically efficient solutions for relevant range of risk aversion†.

	Activity levels					
Risk aversion	Barley (ha)	Corn (ha)	Sugar beet (ha)	Sunflower (ha)	Wheat (ha)	ETNR§ (1000 Rials)
0.00015	1.30	0.00	1.35	0.00	3.30	6545.00
0.000085	1.15	0.23	1.55	0.00	3.05	6582.00
0.000065	0.86	0.25	1.67	0.40	2.83	6789.00
0.000045	0.80	0.38	1.80	0.54	2.60	6832.00
0.000025	0.00	0.85	2.10	1.28	1.90	7548.00
0.000002	0.00	0.97	2.34	1.32	1.50	7615.00

† The representative farm has 6.5 ha of operated land.

§ ETNR stands for expected value of total net revenue.

Also, tradeoffs between economically efficient cropping pattern and the risk aversion coefficient for the representative farm are given in Table 2. It illustrates that the optimal crop combination is sensitive to variations in risk preference. As r_A decreases farmers select a farm plan which contains more high net return cash crops (i.e., corn, sugar beet and sunflower). From Table 2 it follows that expected value of total net return (ETNR) increases as aversion to risk decreases.

The results of this study demonstrated that risk aversion plays an important role in farmers' behavior. Thus, ignoring this factor can lead to select programs which overestimate their levels of total net revenue. This may explain why results of deterministic models which are based on profit maximization and assumed risk neutrality are often disregarded by farmers as unrealistic.

Using UEP model, it was illustrated that this procedure can effectively incorporate the SDWRF criterion in whole-farm planning. This procedure can develop an efficient set for the relevant range of absolute risk aversion. This is especially useful whenever in complete information is available about farmers' risk attitudes. However, it needs specific information on the lower and upper bounds for a decision maker's coefficient of absolute risk aversion.

The conclusions reached in this study are, mainly, based on the interpretation of the results of the representative farm analysis, with the assumption that the selected farm was reasonably representative. While the sample farm selected was the median farm, with all the farms located in a limited number of villages in the study region, there still remains variability within the group. The farm modeled may not therefore provide a close representation of all the sample farms and the conclusions of this study should therefore be interpreted with reservations.

LITERATURE CITED

1. Anderson, J.R. 1974. Risk efficiency in the interpretation of agricultural production research. Rev. Market. Agric. Econ. 42:131-184.
2. Anderson, J.R., J.L. Dillon and J.B. Hardaker. 1977. Agricultural Decision Analysis. Iowa State University Press. Ames, IA, U.S.A. 344 p.
3. Anderson, J.R. and W.E. Griffiths. 1982. Production risk and efficient allocation of resources. Aust. J. Agric. Econ. 26:226-232.
4. Anderson, J.R. and J.L. Dillon. 1992. Risk Analysis in Dryland Farming Systems. FAO. Farm System Management Series 2. Rome, Italy. 180 p.
5. Arrow, K.J. 1965. Aspects of the Theory of Risk-Bearing. Academic Bookstore. Helsinki, Finland. 75 p.
6. Brooke, A., D. Kendrick and A. Meeraus. 1988. GAMS: General Algebraic Modeling System References. User's Guide. The Scientific Press. Redwood City. CA, U.S.A. 289 p.
7. Charnes, A. and W.W. Cooper. 1959. Chance constrained programming. Manage. Sci. 6:73-79.
8. Cochran, M.J., L.J. Robison and W. Lodwick. 1985. Improving the efficiency of stochastic dominance techniques using convex set stochastic dominance. Amer. J. Agric. Econ. 67:289-295.
9. Crocomo, C.R. 1979. Risk efficient fertilizer rates: an application to corn production in Brazil. Ph.D. thesis, Michigan State University, East Lansing, MI, U.S.A.
10. Da Cruz, E.R. and V.H. Da Fonseca Porto. 1988. Simplified risk analysis in agricultural extension. Agric. Econ. 1:381-390.

11. Dillon, J.L. and J.B. Hardaker. 1980. Farm Management Research for Small Farmer Development. FAO. Rome, Italy. 302 p.

12. Ellsberg, D. 1961. Risk, ambiguity and the Savage axioms. Quarterly Econ. 75:643-669.

13. Flood, M., F. McCamley and K. Schneeberger. 1985. Restricted absolute risk-aversion stochastic dominance. Agric. Econ. Report No. 1985-6. Department of Agricultural Economics. University of Missouri, Columbia, MO, U.S.A.

14. Friedrich, K.H. 1977. Farm Management Data Collection and Analysis: An Electronic Data Processing, Storage and Retrieval System. FAO Agricultural Services Bulletin No. 34. Food and Agriculture Organization of the United Nations. Rome, Italy. 160 p.

15. Ghatak, S. and K. Ingersent. 1984. Agriculture and Economic Development. Brighton Wheatsheat Books. Brighton, England. 380 p.

16. Goh, S., S. Shih, M.J. Cochran and R. Raskin. 1989. A generalized stochastic dominance program for the IBM PC. South. J. Agric. Econ. 21:175-182.

17. Halter, A.N. and G.W. Dean. 1965. Use of simulation in evaluating management policies under uncertainty: An application to a large scale ranch. J. Farm Econ. 47:557-573.

18. Hammond, J.S. 1974. Simplifying the choice between uncertain prospects where preference is nonlinear. Manage. Sci. 20:1047-1072.

19. Hardaker, J.B., S. Pandey and L.H Patten. 1991. Farm planning under uncertainty. Rev. Market. Agric. Econ. 59:9-22.

20. Harris, T.R. and H.P. Mapp. 1986. A stochastic dominance comparison of water-conserving irrigation strategies. Amer. J. Agric. Econ. 68:298-305.

21. Hazell, P.B.R. and R.D. Norton. 1986. Mathematical Programming for Economic Analysis in Agriculture. Macmillan. New York, U.S.A. 400 p.

22. King, R.P. and L.J. Robison. 1981. An interval approach to the measurement of decision maker preferences. Amer. J. Agric. Econ. 63:510-520.

23. Kramer, R.A. and R.D. Pope. 1981. Participation in farm commodity programs. Amer. J. Agric. Econ. 63:119-128.

24. Meyer, J. 1977a. Choice among distributions. J. Econ. Theory 14:326-336.

25. Meyer, J. 1977b. Second degree stochastic dominance with respect to a function. Int. Econ. Rev. 18:477-487.

26. Patten, L.H., J.B. Hardaker and D.J. Pannell. 1988. Utility-efficient programming for whole-farm planning. Aust. J. Agric. Econ. 32:88-97.

27. Pratt, J.W. 1964. Risk aversion in the small and in the large. Econometrica 32:122-136.

28. Rae, A.N. 1977. Crop Management Economics. William Clowes. London, England.

29. Raiffa, H. 1968. Decision Analysis: Introductory Lectures on Choice under Uncertainty. Addison-Wesley. Reading. MA, U.S.A. 295 p.

30. Robison, L.J. 1982. An appraisal of expected utility hypothesis tests constructed from responses to hypothetical questions and experimental choices. Amer. J. Agric. Econ. 64:367-375.

31. Roy, A.D. 1952. Safety first and the holding of assets. Econometrica 20:431-449.

32. Schoemaker, P.J. 1982. The expected utility model: its variants, purposes, evidence and limitations. J. Econ. Lit. 20:529-563.

33. Telser, L. 1955. Safety-first and hedging. Rev. Econ. Stud. 23:1-16.

34. Whitmore, G.A. 1970. Third-degree stochastic dominance. Amer. Econ. Rev. 60:457-459.

35. Yitzhaki, S. 1982. Stochastic dominance, mean variance, and Gini's mean difference. Amer. Econ. Rev. 72:178-185.