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NOTE

ESTIMATION OF PRODUCTION FUNCTIONS INCORPORATING BIOTECHNOLOGY FOR MILK, CHEESE AND FAT

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ABSTRACT

This study represents a specification search for production functions which incorporate biotechnology. A linear, a Cobb-Douglas, a quadratic, and a transcendental function for milk, cheese and fat, incorporating bovine growth hormone were fitted to the data set. The results indicated a statistically positive and significant relationship between bovine growth hormone and the production of milk, cheese, and fat. The analysis of the estimated average physical products indicated an increase in the amount of feed and the feed efficiency, shown by an increase in the average physical products per unit of dry matter intake.

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تخمین توابع تولیـد بـا بکـارگیری بیوتکنولـوژی بـرای شـیر، پنیر و چربی

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چکیده

در این مقاله، موضوع جستجو برای تعیین فرم توابع تولیدی کـه در آن هـا بیوتکنولـوژی بـه کار رفته باشد، مورد بررسی قرار گرفـت. توابـع تولیـد خطـی، کـاب-داگـلاس، درجـه دوم، و متعالی برای شیر، پنیر و چربی با توجه به استفاده از هورمون رشـد گـاوی، بـه مجموعـه داده هـا برازش شد. نتایج نشان داد که رابطه معنی داری بین هورمون رشد گاوی و تولید شیر، پنیر وچربی وجود دارد. تحلیل توابع متوسط تولید به دست آمده نشانگر افزایش مقدار مصـرف غـذا و ضریـب تبدیل آن به وسیله افزایش متوسط تولید هر واحد از ماده خشک مصرفی می باشد.

INTRODUCTION

The development of biotechnology such as bovine somatotropin (BST) might increase milk production per cow and lower costs per unit of milk produced. Biotechnology research can have major impacts on food production with substantial economic consequences for producers and consumers. The development and adoption of new biotechnologies can have an impact on the individual cost-and-returns structures, the price and supply relationships that affect all producers of commodities, and the availability of the commodities to society as a whole. All of these have policy implications.

In this study, four different functional forms are fitted to the data set: a linear function, a Cobb-Douglas type or log-linear, a quadratic, and a transcendental function. These models may not represent the true functional form characterizing the production of milk, cheese, or fat, because that is not known. However, they could be used as part of a specification search in which other functional forms migh be fitted to the data. In general, economic theory and animal (biological) science can be used as sources of this specification search (5).

Background

Production-form selection is a crucial part of applied research in production analysis because the "true" functional form of any relationship is not known *a priori*. Economic theory usually provides the basic tools and guidance for the specification search. Another reasonable source of specification is animal (biological) science, which also provides some of the characteristics of any relationship. If theory does not provide the proper guidance, wrong conclusions and invalid hypotheses tests might result (5, 6).

Economic theory specifies the regression variables and the signs of the coefficients of the variables, but not the functional form of the regression equation, because, in reality, no "true" model could ever hope to be found and even economic theory sometimes provides no basis at all for choosing among alternative forms (4, 6).

Some considerations can be taken into account when choosing functional forms in an attempt to find one that fits the data. Fuss *et al.* (3) list the following general criteria for that purpose: 1- Parsimony in parameters-The functional forms should contain only the amount of parameters needed to maintain consistency with the maintained hypotheses. 2- Ease of interpretation-The functional forms chosen should contain parameters which have intrinsic and economic interpretations. 3- Computational ease-There are less computation costs associated with linear-in-parameters systems. 4-

Interpolative robustness- The chosen functional form should be well behaved and should display consistency with the maintained hypotheses within the range of the observed data. 5- Extrapolative robustness-The chosen functional form should take into consideration forecasting applications.

Griffin *et al.* (4) also list criteria for choosing how one functional form may be deemed more appropriate than another, but they take a more specific approach than Fuss *et al.* (3). They grouped the criteria according to whether they relate to maintained hypotheses, estimation, data, or application: 1- Maintained hypotheses-The function can be called appropriate if the maintained hypotheses implied by it are acceptable. 2- Estimation-Data availability, data properties, and the availability of computing resources can affect the choice of functional form for statistical estimation. 3- Data-The choice of functional form for data-specific considerations; i.e., goodness-of-fit and general conformity to data, requires the use of a specific data set and the findings are not general. 4- Application-The number of potential criteria within this category is quite large and highly customized but the authors give the example that if the resulting equation is to be used in simulation or optimization procedures, there may be other desirable properties for functional forms.

It should be noted that the choice of any particular functional form immediately imposes untestable (maintained) hypotheses on the empirical analysis. Maintained hypotheses are not themselves testable from the data set as part of the analysis, but are assumed true once the production functional form is chosen. Much of the discussion by Griffin *et al.*(4) centers around maintained hypotheses. They also address how "less restrictive functional forms would always be desirable for the greater information needed to adequately specify such relationships". Fuss *et al.* (3) also discuss the issue of flexibility in regard to maintained hypotheses. Debertin *et al.* (2) point out that, "Because reductions in maintained hypotheses come at a cost, added flexibility is not always desirable, and there are likely to be cost- effective opportunities to achieve particular dimensions of flexibility".

Another important idea in the choosing of functional forms is the incorporation of technical progress in the model. The selection of functional forms incorporating the effects of a biotechnological advance shares all of the points made above. Biotechnology can be treated as a new technology which has long been an interest of agricultural economists. In economic theory, new technology has been defined as capital using if it increases the marginal rate of technical substitution of capital for labor, which means an increase in the capital-labor ratio at a given level of output, or an increase in the capital- output ratio. If the opposite of this is true then new technology is defined as labor using. If it does not change the marginal rate of technical substitution or the capital-labor ratio or the capital-output ratio, then it is defined as neutral. For an explicit assumption of technical progress, Debertin (1) suggests Cobb-Douglas specification which implies that the Cobb-Douglas could be used for a model incorporating biotechnology.

MATERIALS AND METHODS

The Models

In order to estimate the production function for milk, cheese, and fat, the following models are fitted to the data set:

$$(1)\ Y=\alpha_0+\alpha_1(DIML)+\alpha_2(DMI)+\alpha_3(COWTYPE)+\alpha_4(TRT)(DMI)+\alpha_5(BST)(DMI)$$

$$(2)\ Y=\beta_0(DIML)^{\beta 1}(DMI)^{\beta 2+\beta 3(TRT)+\beta 4(BST)}\ EXP\ [\beta_5\ (COWTYPE)]$$

$$(3)\ Y=\gamma_0+\gamma_1(DIML)+\gamma_2(DMI)+\gamma_3(COWTYPE)+\gamma_4(TRT)(DMI)+\gamma_5(BST)(DMI)+\gamma_6(DIML)^2+\gamma_7(DMI)^2+\gamma_8(DIML)(DMI)$$

$$(4)\ Y=(DIML)^{\mu 0}(DMI)^{\mu 1+\mu 2(TRT)+\mu 3(BST)}EXP[\mu_4+\mu_5(COWTYPE)+\mu_6(DIML)+\mu_7(DMI)]$$

Model description. The description of the models are as follows: The first model is a linear function, the second one is a Cobb-Douglas type, or log-linear, the third one is a quadratic, and the forth one is a transcendental function. A linear production function has constant marginal physical product (MPP) for all inputs. A linear production function has no maxima, which implies that output can be increased at a constant rate indefinitely by increasing the amount of the inputs. The inclusion of the linear function in this study is merely for the purpose of comparison with other forms.

The Cobb-Douglas function, with $0<\beta i<1$, has decreasing MPP and APP for all inputs. Also, it has unitary elasticity of substitution and convex isoquants. If one input is increased indefinitely while the level of use of the other inputs remains constant, the marginal physical product of the increased input will approach zero asymptotically and output will increase without limit. Hence there is no such thing as output maximization because the first order necessary conditions do not hold. But, profit maximization is possible. The Cobb-Douglas function is transformed into its log-linear form, so that it can be estimated by OLS which is the best linear unbiased estimator (BLUE)(7).

The quadratic form is inherently additive rather than multiplicative. It is appealing because it gives a second-order local approximation to any function. It is linear in its paramerers, thus it can be estimated via OLS, but it is messy to mathematically manipulate.

The transcendental function can be viewed as a modification of a Cobb-Douglas type function that can depict the three stages of production and has variable production elasticities, yet at the same time retain a function that was related to the Cobb-Douglas. Like the Cobb-Douglas, the transcendental can also be transformed to natural logs with the resulting equation that is linear in the parameters and is again easily estimated via OLS.

The dummy variables of the model are set up in a way that the interactions of the different rations (TRT) and the bovine growth hormone (BST) with dry matter intake (DMI) will only affect the slope of the production functions and not the intercepts. However, the COWTYPE variable is set up so that it can only affect the intercepts.

Variable definition. The definitions of the variables are as follows: The dependent variable milk (MILK) is the amount of milk, in total kg produced by the cow during the lactation period. Cheese (CHEESE) and fat (FAT) as joint products of milk processing are also used as dependent variables in this study. The amount of cheese and fat, in kg, are obtained from the milk produced from each cow.

Days in milk (DIML) is hypothesized to have a positive effect on the production of milk, cheese, and fat. The more days in milk, the more the production of the three joint products. Increasing the amount of milk produced is the only reason for milking the cow longer.

Cow type (COWTYPE) is a dummy variable which is equal to one if the cow is multiparous and equals zero if the cow is primiparous. It is obvious from Table 1 that multiparous cows produce more milk than do primiparous cows. Therefore, from the way the dummy variable is set up, cow type is hypothesized to have a positive effect on the production of milk, cheese, and fat. Dummy variable treatment (TRT) with the value equals 1 if the ration is treatment 1 and equals zero if the ration is treatment 2. Again from Table 1, cows with treatment 1 produce more milk than cows with treatment 2. Therefore, the dummy treatment is hypothesized to have a positive impact on the production of milk, cheese, and fat.

Bovine somatotropin (BST) is the most important dummy variable in this study. The dummy BST equals one if the cows are injected with BST, and equals zero if the cows are injected with placebo. BST is expected to increase milk production per cow by 10-15 percent, and feed efficiency by 5-10 percent. Therefore the dummy BST is hypothesized to have a positive effect on the production of milk, cheese, and fat.

Dry matter itake (DMI) is the amount of feed intake, in kg, eaten by the cow during the lactiation period. Dry matter intake is hypothesized to have a positive impact on the production of milk, cheese, and fat. Cows treated with BST are hypothesized to eat more than cows not treated by BST, because nutrient demand for the cow would increase as the cow produces more milk due to BST use.

The Data

The data used in this study are based on the raw data from a designed experiment conducted by the USDA and the University of Wisconsin. The experiment used 32 primiparous and 32 multiparous Holstein cows that were randomly assigned at parturition to one of two diets differing in energy content and randomly assigned to either a placebo or a recombinant BST group. Eight cows from each dietary treatment group were assigned to each injection treatment.

Daily injections of saline or BST started at the beginning of week 13 of lactation and continued through week 43. Feed intake for individual cows was measured by recording daily feed offered and feed refused. For the purpose of this study the data are rearranged and since the number of observations is not too large (n=64), the entire data set is included. Table 1 indicates the minimum, the maximum and the mean values for each variable categorized by cow type, treatment and BST.

Table 1. Minimum, maximum, mean, and standard deviation for each variable categorized by cow type, treatment, and BST.

Variable	n	Minimum	Maximum	Mean	SD
		Multiparous, treatment 0, no BST			
MILK	8	5094.0000	9621.0000	7278.1300	1479.1000
CHEESE	8	583.0000	952.0000	734.8750	118.2388
FAT	8	200.0000	332.0000	261.2500	41.1469
DIM	8	259.0000	308.0000	287.8750	18.8713
DMI	8	5486.6000	7076.3000	6173.3900	591.6680
		Multiparous, treatment 0, BST			
MILK	8	7801.0000	10158.0000	8803.3700	962.0797
CHEESE	8	762.0000	1070.0000	883.3750	106.9832
FAT	8	269.0000	399.0000	326.3750	44.7148
DIM	8	280.0000	308.0000	302.7500	9.7211
DMI	8	6568.8000	7912.8000	7088.0300	484.7142
		Multiparous, treatment 1, no BST			
MILK	8	7009.0000	9390.0000	8495.8700	809.2368
CHEESE	8	735.0000	955.0000	825.7500	85.2270
FAT	8	243.0000	336.0000	285.5000	34.9898
DIM	8	266.0000	308.0000	292.2500	20.0624
DMI	8	5157.7000	7611.8000	6259.1400	789.2168

Table 1. (Continued)

		Multiparous, treatment 1, BST			
MILK	8	8243.0000	13244.0000	10283.0000	1561.0000
CHEESE	8	842.0000	1148.0000	1001.3700	91.3907
FAT	8	307.0000	392.0000	349.0000	24.5415
DIM	8	273.0000	308.0000	301.8750	12.6541
DMI	8	6381.2000	7940.1000	7057.4900	504.7998
		Primiparous, treatment 0, no BST			
MILK	8	5074.0000	7838.0000	6526.6300	902.8430
CHEESE	8	560.0000	752.0000	648.8750	68.0660
FAT	8	200.0000	267.0000	234.1250	26.1230
DIM	8	266.0000	308.0000	297.5000	17.5499
DMI	8	5410.8000	5847.8000	5348.6100	439.8643
		Primiparous, treatment 0, BST			
MILK	8	5778.0000	7976.0000	6914.6300	784.4385
CHEESE	8	599.0000	802.0000	720.3750	65.2182
FAT	8	209.0000	303.0000	265.0000	30.9654
DIM	8	280.0000	308.0000	297.5000	12.9615
DMI	8	4463.8000	6444.2000	5675.8600	670.0809
		Primiparous, treatment 1, no BST			
MILK	8	5549.0000	8340.0000	7160.0000	933.1904
CHEESE	8	637.0000	893.0000	746.5000	101.5241
FAT	8	219.0000	340.0000	266.0000	46.2292
DIM	8	280.0000	308.000	296.6250	12.3744
DMI	8	5059.6000	6471.5000	5583.1100	455.2966
		Primiparous, treatment 1, BST			
MILK	8	6839.0000	9724.0000	8161.5000	845.9912
CHEESE	8	736.0000	915.0000	813.3750	60.3465
FAT	8	258.0000	314.0000	291.8750	17.0414
DIM	8	287.0000	308.0000	300.1250	9.4934
DMI	8	5524.4000	6017.2000	5857.6000	167.8737

RESULTS AND DISCUSSION

The estimated production functions for milk, cheese and fat are summarized in Tables 2, 3 and 4.

Table 2. The results of production functions estimations for milk†.

Linear model	Cobb-Douglas model	Quadratic model	Transcendental model
α_0=1516.62(0.72)	β_0=2.35(0.53)	γ_0=-38943.86 (-0.70)	μ_0=15.35(1.09)
α_1=-2.69(-0.91)	β_1=-0.22(-0.68)	γ_1=220.97(0.56)	μ_1=0.69(0.54)
α_2=1.30(5.50)	β_2=1.06(5.87)	γ_2=3.79(1.13)	μ_2=0.01(4.63)
α_3=52.68(0.17)	β_3=0.01(0.15)	γ_3=182.74(0.55)	μ_3=0.01(1.61)
α_4=0.17(5.11)	β_4=0.01(4.71)	γ_4=0.17(5.18)	μ_4=-68.92(-1.03)
α_5=0.07(1.95)	β_5=0.01(1.77)	γ_5=0.07(1.94)	μ_5=0.02(0.47)
		γ_6=-0.21(-0.29)	μ_6=0.05(-1.10)
		γ_7=0.00(1.33)	μ_7=0.00(0.26)
		γ_8=-0.02(-1.26)	
F-value=35.46	F-value=31.96	F-value=22.38	F-value=22.75
R-squared=0.75	R-squared=0.73	R-squared=0.77	R-squared=0.74

† Figures in parentheses are t-values.

Linear model. Statistically, linear model gives high F values and R^2s. MPP is always positive for any positive level of input use. The slope and the curvature of MPP are zero, hence MPP is constant with no curvature. The sign, slope, and curvature of APP are identical to those of MPP which imply that APP is constant with no curvature. This model does not conform with theory in the sense that production would increase indefinitely as the amount of inputs are increased.

Table 3. The results of production functions estimations for cheese†

Linear model	Cobb-Douglas model	Quadratic model	Transcendental model
α_0=686.49(4.31)	β_0 =2.41(2.03)	γ_0=-3873.63(-0.92)	μ_0=13.30(1.29)
α_1 =-2.58(-4.08)	β_1 =-0.93(-3.94)	γ_1 =24.43(0.83)	μ_1 =0.65(0.71)
α_2 =0.13(7.53)	β_2 =1.09(8.17)	γ_2 =0.35(1.40)	μ_2 =0.01(5.58)
α_3 =-24.88(-1.07)	β_3 =-0.03(-1.02)	γ_3 =-13.24(-0.53)	μ_3 =0.01(2.85)
α_4 =0.01(5.76)	β_4 =0.01(5.63)	γ_4 =0.01(5.77)	μ_4=-60.67(-1.25)
α_5 =0.01(3.03)	β_5 =0.01(3.06)	γ_5 =0.01(3.01)	μ_5 =-0.02(-0.56)
		γ_6 =-0.03(-0.63)	μ_6 =-0.05(-1.38)
		γ_7 =0.00(0.86)	μ_7 =0.00(0.43)
		γ_8 =-0.00(-1.13)	
F-value=50.62	F-value=48.06	F-value=31.72	F-value=34.72
R-squared=0.81	R-squared=0.80	R-squared=0.82	R-squared=0.81

† Figures in parentheses are t-values.

Table 4. The results of production functions estimations for fat†.

Linear model	Cobb-Douglas model	Quadratic model	Transcendental model
α_0= 101.36(1.61)	β_0=-1.63(-1.24)	γ_0=-831.36(-0.49)	μ_0= 9.69(0.84)
α_1=-0.45(-1.76)	β_1=-0.45(-1.74)	γ_1= 4.40(0.37)	μ_1= 1.24(1.20)
α_2= 0.05(7.05)	β_2= 1.12(7.66)	γ_2= 0.12(1.21)	μ_2= 0.01(3.39)
α_3=-13.88(-1.50)	β_3=-0.05(-1.51)	γ_3= 11.75(1.17)	μ_3= 0.01(2.83)
α_4= 0.00(3.44)	β_4= 0.01(3.51)	γ_4= 0.00(3.36)	μ_4=-49.86(-0.92)
α_5= 0.00(2.87)	β_5= 0.01(2.96)	γ_5= 0.00(2.83)	μ_5= 0.04(-1.12)
		γ_6= -0.00(-0.25)	μ_6=-0.03(-0.88)
		γ_7= 0.00(0.22)	μ_7= 0.00(-0.15)
		γ_8=-0.00(-0.69)	
F-value=39.96	F-value=39.44	F-value=24.16	F-value=27.70
R-squared=0.78	R-squared=0.77	R-squared=0.78	R-squared=0.78

† Figures in parentheses are t-values.

Cobb-Douglas model. The derivatives of APP and MPP with respect to DMI are the slopes of APP and MPP, respectively, and are positive. The curvatures of APP and MPP are the second derivatives of APP and MPP, respectively, and are negative; which means that both APP and MPP are increasing at a decreasing rate (1). The value of APP is less than the value of MPP which indicates that the production function is in stage I.

Quadratic model. The sign and slope of MPP for the quadratic model are positive, while its curvature is zero, which implies that MPP is increasing at a constant rate for all inputs. The sign of APP is positive, its slope is negative for lower values of dry matter intake and increases as dry matter intake increases, while its curvature is negative. The APP has an unacceptable shape based on production theory. This model gives high R^2s and low F-values compared to the other model.

Transcendental model. The results indicate that this model compared to log-linear model gives a slight improvement in R^2s, but with the cost of much smaller F-values. The sign, slope and curvature of MPP in this model for milk and cheese are positive which implies that MPP is increasing at an increasing rate. The sign of APP for milk and cheese is always positive, however, its slope is negative for lower values of dry matter intake and increases as dry matter intake increases. This is not an acceptable shape of APP according to production theory. The sign of MPP for fat is positive, while its curvature is negative. The shape of MPP for fat is positivw, while its curvature is negative. The shape of MPP in this case is not acceptabe. The sign of APP and its slope for fat are positive but its curvature is negative which implies that APP is increasing at a decreasing rate.

CONCLUSIONS

In this study four of the twenty functional forms reviewed by Griffin *et al.* (4) were selected to represent the production of milk, cheese, and fat, incorporating biotechnology. Each functional form has its own implications from a theoretical as well as statistical perspective. A linear model gives high F values and R^2s but it has the implications of constant MPP and APP. Statistically, a log-linear model is inferior compared to linear model. Yet, a log-linear model gives MPP and APP which are increasing at a decreasing rate, implying stage I production. Transcendental model produces a slightly higher R^2s than log-linear, but the improvement in R^2s comes with a cost in the form of lower F values and the insignificance of important explanatory variables. Furthermore, neither neoclassical nor contemporary theories recognize the shape of APP and MPP which is decreasing and then increasing. Quadratic model gives an even worse result despite the highest R^2s among the models. Quadratic model also produces unacceptable MPP and APP. Further research might be interested in selecting functional forms in production analysis that incorporate biotechnological advance from the rest of the list given by Griffin *et al.*(4), or other related sources.

This study also indicates that BST increases the production of milk, cheese, and fat and feed efficiency of the dairy cow. From the average physical product analyses, the impact of BST in increasing the production of milk, cheese, and fat is not only felt through the increase in the amount of feed but also through the feed efficiency as indicated by the increase in the average physical products per unit of dry matter intake.

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