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EVALUATION OF FRACTAL DIMENSIONS FOR ANALYSIS OF AGGREGATE STABILITY

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ABSTRACT

Fractal theory is increasingly being used to characterize aggregate size distributions of soils. Our purpose was to compare indirect number-size fractal dimension (D_n, a fitting parameter), mass-size fractal dimension (D_m), and mean weight diameter (MWD), as measures of soil aggregate stability. Samples of a calcareous silty clay soil (Calcixerollic Xerochrept) was treated with cationic, anionic and clay emulsion petroleum mulches and "krilium merloam" (krilium) which is a copolymer of vinyl acetate and maleic acid. Application amounts were 0, 0.5, 1.0 and 1.5 g kg^{-1} on dry mass basis. Assuming cubic aggregates of constant dry density, D_n was estimated from cumulative size-frequency distribution data plotted on a log-log scale and D_m was estimated from cumulative mass-size distribution data in the same manner. D_n and D_m decreased with increasing amount of mulch applications indicating an increase in aggregate stability as a result of the addition of the mulch. The increase was most pronounced for the krilium, indicating that this chemical was most effective for increasing aggregate stability. Application of MWD, D_n and D_m may lead to different conclusions in the assessment of the effectiveness of the various chemicals for improving soil aggragate stability. For less effective chemicals (petroleum mulches) D_n values (a fitting parameter) were a better indicator of differences in fragment

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distributions, while for more effective chemicals (i.e., krilium) MWD showed the differences in fragment distribution better than D_n and D_m. Application of krilium at amounts greater than 0.5 g kg^{-1} might have resulted in aggregates with complete fragmentation properties, while the aggregates of other treatmentes did not show this behavior. The D_n (a fitting parameter) may be used to describe the fractal behavior of the soil fragments with MWD greater than 0.67 and 1.17 mm for petroleum mulch and krilium applications, respectively. However, mass-size fractal dimensions failed to show the effects of various rates of petroleum mulches on the fragment size distributions.

Key words: Petroleum mulches, Soil conditions.

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ارزیابی ابعاد فراکتالی برای تعیین ثبات خاکدانه

علیرضا سپاسخواه ، سید علی اکبر موسوی و لاری بورزما

به ترتیب استاد و استادیار بخش آبیاری دانشکده کشاورزی دانشگاه شیراز، شیراز، جمهوری اسلامی ایران و استاد بخش علوم گیاه- خاک دانشگاه ایالتی اورگن، آمریکا.

چکیده

کاربرد نظریه فراکتالی برای تعیین توزیع اندازه خاکدانه ها روزبروز افزایش می یابد. در این پژوهش، بعد فراکتالی غیر مستقیم حاصل از اندازه تعداد خاکدانه (D_n به عنوان عامل برازش)، بعد فراکتالی اندازه -جرم(D_m) و میانگین وزنی قطر (MWD) خاکدانه به عنوان عوامل تعیین کننده ثبات خاکدانه مقایسه گردید. خاکی آهکی با بافت سیلتی رسی (کالسی زرولیک- زروکرپت) با امولسیون قیر کاتیونی، آنیونی، رسی و ماده شیمیایی 'کریلیوم مرلوم' که یک پلیمرمشترک وینیل استات و

مالئیک اسید (خاکپوش) است تیمار گردید. مقدار مواد به کار رفته عبارت بودند از صفـر، ۰/۵، ۱/۰ و ۱/۵ گرم بر کیلوگرم جرم خشک خاک. با فرض خاکدانـه هـای مکعبـی شـکل و چگـالی خشـک ثـابت، مقادیر D_n از رسم داده هـا ی توزیـع تجمعـی انـدازه خاکدانـه هـا روی محـور مختصـات لگـاریتم - لگاریتم تخمین زده شد و مقـادیر D_m از رسـم داده هـای توزیـع تجمعـی جـرم خاکدانـه روی محـور مختصات لگاریتم- لگاریتم به دست آمد. مقادیر D_n و D_m با افزایش مقدار کاربرد خاکپوش ها کاهش یافت که نشان دهنده افزایش ثبات خاکدانه ها به خاطر کاربرد خاکپوش ها مـی باشـد. افزایـش ثبـات خاکدانه ها برای کریلیوم نمایان تر بود و نشان داد که کریلیوم در ثبات خاکدانه ها بیشترین تـاثیر را داشته است. کاربرد معیارهای مختلف تعیین ثبات خاکدانـه هـا یعنـی MWD، D_n و D_m ممکـن اسـت نتایج متفاوتی برای خاکپوش ها به بار آورد. برای خاکپوش هـا ی کـم اثـر (خـاکپوش هـای قـیری) معیار D_n (به عنوان عامل برازش) معرف بهتری برای نشان دادن تفاوت های بین توزیع خاکدانه ها است . در حالی که برای خاکپوش پر اثر (کریلیوم) معیار MWD تفاوت های بین توزیع خاکدانه ها را بهتر از معیار های D_n و D_m نشان داد. کاربرد کریلیوم با مقدار بیش از ۰/۵ گرم بر کیلـو گـرم خـاک خشک موجب تولید خاکدانه هایی با خواص تقسیم پذیری کامل شد در حالی که خاکدانه های حـاصل از خاکپوش های دیگر تقسیم پذیری کاملی را نشان نداد. بنـابراین معیـار D_n (عـامل بـرازش) ممکـن است که برای توصیف خاصیت فراکتالی خاکدانه هـا بـا MWD بیـش از ۰/۶۷ و ۱/۱۷ میلـی مـتر بـه ترتیب برای خاکپوش های قیری و کریلیوم به کار برده شود. هم چنین معیار D_m نتوانست اثر مقادیر مختلف خاکپوش های قیری را بر ثبات خاکدانه ها و ایجاد تفاوت در توزیع اندازه خاکدانـه هـا نشـان دهد.

INTRODUCTION

Several numerical procedures have been proposed for characterization of soil aggregate-size distributions. Fractals seem to be useful for describing properties of porous media, particularly soil structure (2, 23, 24, 27, 28, 32). Fractal parameters have increasingly become more important for quantifying and describing aggregate size distribution of soils. Its use in characterization of aggregate size distribution provides a quantitative description of soil structure and its stability. The results can also be used to evaluate effects of soil conditioners on the stability of soil structure (16).

Fractal parameters can demonstrate specific dependencies of measurable properties on the scale of measurements. For granular materials, these dependencies can be represented by power law relationships between mass m and radius R, porosity n and R, and surface area A and R (26). The general equation is $y=x^D$ where y is the mass fraction, porosity, or surface area, x is the radius, and D is the fractal dimension. The fractal dimension is thus a scaling parameter. In general, fractal dimensions are positive numbers ranging from 0 to 3, with fractal behavior corresponding to $0<D_m<3$, and $0<D_n<3$ for mass and volume, respectively, and $0<D_s<2$ for surface area (24). The power law is valid in a range of radii between so-called upper and lower cutoffs R_{max} and R_{min}. The ratio R_{max}/R_{min} theoretically must not be less than $2^{1/D}$

Several soil properties have been shown to be fractal. Perfect and Kay (17) indicated that fractal cubes may provide a more realistic representation of soil aggregates. Particle size distributions were shown to be fractal by Tyler and Wheatcraft (27, 28), Bartoli *et al.* (2), Niemeyer and Ahl (14), and Wu *et al.* (31). Fractal scaling was demonstrated for soil aggregates by Young and Crawford (32), Perfect and Kay (16), Perfect *et al.* (18) and Rasiah *et al.* (20). Eghball *et al.* (6) characterized soil fragmentation due to several tillage methods and cropping sequences using fractals. Mass-size relationships were shown to be identical to direct number-size fractal relationships for soil particles given in a specific range (19, 31). However, Crawford *et al.* (5) showed that there may be differences between the number-size fractal and mass-fractal dimensions. Rieu and Sposito (23) showed that a combination of fractal particles and fractal aggregates was a useful schematization of soil structure. Pachepsky *et al.* (15) showed that the fractal dimension, D, of pore surface area significantly increased due to cycles of wetting and drying. Perfect and Kay (17) postulated that it may be possible to predict the pore-size distribution of soil aggregates from brittle fracture data. Brakensiek and Rawls (3) and Rasiah *et al.* (20) showed that the content of clay and sand particles in soil could be used to predict fractal dimensions in pore number vs. size and aggregate mass vs. size dependencies. Bartoli *et al.* (2) and Brakensiek *et al.* (4) noted that soil microporosity and macroporosity appear to be fractal. Logsdon (8) investigated a possible relationship between hydraulic conductivity and the fractal dimension of aggregates.

Fractal parameters have been used recently to evaluate the application of oily waste for improvement of soil aggregate stability (21). Van Bavel (29) used the mean weight diameter (MWD) and Mazurak (11) used the geometric mean diameter (GMD) to integrate aggregate-size distributions obtained by mechanical sieving. These methods were found to be biased towards the large aggregates (MWD) or towards the smaller aggregates (GMD) (1). A less biased estimate of the water stable aggregate size distribution (WSASD) was obtained by employing power function to describe the relation between the cumulative percentage by mass of aggregates less than a given diameter, $W_{<x}$, and aggregate diameter, x (1). the power function has the following form:

$$W_{<x}=A(x)^{B} \qquad [1]$$

where A and B are constants. The magnitudes of A, and B are determined by logarithmic transformation of the WSASD data and linear regressoin of $\ln w_{<x}$ on $\ln x$ using the method of least squares. Baldoc and Kay (1) used the B coefficient as the index of aggregate-size distribution.

Mandelbrot (10) defined the fractal dimension by a power-law function between number and size of objects as follows:

$$N_{>x}=k(x)^{-D} \qquad [2]$$

where N>x is the cumulative number of objects greater than x and k is a constant which is equal to $N_{>x}$ at x=1. D is the fractal dimension which depends on the shape of objects within the distribution, and the overall extent of fragmentation. D can be calculated using number of particles in each size class (D_n) or mass of particles in each size class (D_m). At D=0, the size distribution is dominated by one infinitely large object (16). An increase in D signifies that the number of small objects has increased at the expense of the large ones. Turcotte (25) reported D values ranging from 1.44

to 3.54 for different fragmented materials. For primary soil particles D was reported to be greater than 3 (27). However, this might have been resulted from the linear fitting procedure (21, 22).

By equating Eqs. [1] and [2] and assuming kD/AB to be constant, D can be estimated as follows (17):

D=3-B **[3]**

$$D=3-B \quad [3]$$

However, Perfect and Kay (17) questioned the assumption that kD/AB is constant. When the shape of aggregates is assumed to be cubical with edges of length x, the number of aggregates, N(x), is obtained by dividing the mass of each fraction, w(x), by the mass of a single aggregate of length, x, according to:

$$N(x)=(w(x))/(\rho_{agg}x^3) \quad [4]$$

where ρ_{agg} is the dry aggregate density. According to Perfect and Kay (16), the magnitude of D appears to be independent of the Euclidean geometry used to describe the shape of the aggregates, i.e., the same results are obtained whether spherical aggregates or cubical aggregates are assumed for Eq.[4].

Moosavi and Sepaskhah (13) used MWD as measure for the effects of different petroleum mulches and krilium on aggregate stability of a silty clay soil, where MWD was computed using the equation:

$$MWD=\sum_{j=1}^{k} w_{(j)}x_j \quad [5]$$

where x_j is the arithmetic mean of the j-1 fraction, j is the sieve opening, and $w_{(j)}$ is the proportion of the total sample mass occurring in that fraction. The present study was initiated to evaluate the original data reported by Moosavi and Sepaskhah (13) by using indirect number-size fractal dimension (D_n) or the indirect mass-size fractal dimension (D_m) and to evaluate whether or not the application of MWD, D_n and D_m, leads to different conclusions in the assessment of the effectiveness of various chemicals for improving soil aggregate stability.

MATERIALS AND METHODS

Soil samples (a silty clay, thermic Calcixerollic Xerochrept) from pots of a greenhouse experiment were used for determination of aggregate stability. The soil was mixed with anionic, cationic, clay emulsion, or krilium mulches at amounts of 0, 0.5, 1.0, and 1.5 g kg^{-1} (mass basis) before putting it in pots to be planted with sugarbeet seeds. General characteristics of petroleum mulches are given in Table 1.

Table 1. Some general properties of the petroleum mulches.

Ingredients and properties	Type of mulch		
	Anionic emulsion	Cationic emulsion	Clay emulsion
Asphalt (kg kg^{-1})	0.50	0.60-0.64	0.48-0.50
Water (kg kg^{-1})	0.45-0.47	0.33-0.40	0.48-0.50
Emulsifier (kg kg^{-1})	0.2-0.03	0.005-0.01	0.02-0.03
Type of emulsifier	Vinsol resin	Fatty amine	Bentonite
pH	10-12	2-3	6-7
Hydrophobicity	Moderate	Moderate	Low

Details of the experimental procedure were given by Moosavi and Sepaskhah (12). After harvesting the plants, roots were carefully removed from the moist and friable soil. To determine mean weight diameter (MWD) of aggregates at wet sieving conditions, the soil was air dried to a gravimetric water content of 25 g kg^{-1} and passed through a 16-mm sieve (30). The wet sieving and soil handling was done carefully so that the soil was not crushed during the sieving process. One hundred gram samples were used in this study. Mean weight diameter (MWD) of aggregates were determined according to the procedure described by the Committee on Physical Analysis (30) and Eq. [5]. The indirect fragment number-size fractal dimension, D_n was calculated using Eq. [2] and the indirect mass-size fractal dimension, D_m, was estimated using Eq [4], assuming a fragment dry density of 1.63 Mg m^{-3} (16). This value was reported for a silt loam soil

which may not be exactly the same as our soil. However, the results of D calculation may not be erroneous since D values are shown to be insensitive to dry density of aggregates (21). The reduction in estimated D values based on variable fragment dry density (1.5-1.77 Mg m^{-3}) for large and small fragments, respectively, according to Eghball *et al.* (6) in comparison with those from an average value of aggregate dry density of 1.63 was at most 4.5%, which is not significant.

RESULTS AND DISCUSSION

Results of the analyses of variance for D_n, MWD, and D_m are summarized in Table 2. In general, MWD was much more variable than D_n. The coefficients of variation (CV=standard deviation/mean) from the analyses of variance were 0.29 and 0.07 for the MWD and D_n indices, respectively. Application amounts had the dominant effect on MWD and D_n, but mulch type had the dominant effect on the MWD and D_m indices. In other words, MWD was approximately equally sensitive to mulch type and amount. The interaction between mulch type and application amount [(M)(A)] was more significant for the MWD and D_m than for the D_n. These results indicated that the MWD and D_m were more sensitive to the type of mulch than the D_n and that D_n and MWD were more sensitive to the application amounts than D_m.

Table 2. The F values from analyses of variance for the indirect number-size fractal dimension (D_n, a fitting parameter), mass-size fractal dimension (D_m), and mean weight diameter (MWD) of different mulches and application amounts.

		Dependent variable		
Source	df	Indirect number-size fractal dimension D_n	Indirect Mass-size fractal dimension D_m	Mean weight diameter MWD
Mulch type (M)	3	13.27**	47.12**	55.60**
Application amount (A)	3	35.44**	11.13**	42.87**
(M)(A)	9	2.58*	6.53**	13.37**
Rep.	2	1.30	0.43	1.07

*** , ** Significant at the 0.05 and 0.01 probability levels, respectively.**

The D_n indicated that the amounts of 1.5, 0.5 and 0.5 g kg^{-1} of anionic, cationic and clay emulsion mulches, respectively, were effective in improving aggregate stability (Table 3). The MWD results indicated that application of anionic mulch was not effective, but cationic and clay emulsion mulches increased the stability at the amounts of 1.0 and 1.5 g kg^{-1}, respectively. Based on the MWD analysis, krilium, a very good aggregate stabilizer, increased the stability of the aggregates at the amounts of 0.5 to 1.5 g kg^{-1}. However, according to the results of the D_n analysis, the stability of aggregates treated with krilium were not different at the amounts of 0.5 and 1.0 g kg^{-1} and at the amounts of 1.0 and 1.5 g kg^{-1}.

Table 3. Mean weight diameter (MWD), indirect number-size fractal demension (D_n, a fitting parameter), and mass-size fractal dimension (D_m), as affected by incorporation of different amounts of petroleum mulches and krilium into the soil.

Amount of application (g kg^{-1})	Anionic emulsion	Cationic emulsion	Clay emulsion	Krilium
		(MWD-mm)		
0.0	0.200 ef*	0.134f	0.153f	1.188ef
0.5	0.398cde	0.307def	0.272def	0.560c
1.0	0.325cdef	0.507cd	0.360cdef	1.274b
1.5	0.401cde	0.462cd	0.505cd	1.754a
		(D_n)		
0.0	4.03ab	4.42a	4.37a	4.32a
0.5	3.65bc	3.65bc	3.79bc	3.20de
1.0	3.62bcd	3.56cd	3.65bc	2.84ef
1.5	3.55cd	3.61bcd	3.39cd	2.58f
		(D_m)		
0.0	2.79ab	2.88a	2.83ab	2.80ab
0.5	2.77ab	2.79ab	2.79ab	2.56c
1.0	2.83ab	2.76b	2.79ab	2.41d
1.5	2.84ab	2.79ab	2.74b	2.33d

*** Means within each part (MWD, D_n, D_m) followed by the same letter are not significantly different at the 5% level of probability by LSD.**

The MWD analysis showed that only krilium at the amounts of 1.0 and 1.5 g kg^{-1} was superior to other mulches, while D_n indicated that krilium increased aggregate stability more than other mulches at all application amounts. Tyler and Wheatcraft (27) reported that the value of D_n for soil primary particles is greater than 3 while D_m of these particles is less than 3 (28). Therefore, the value of D_n for stable aggregates should be less than 3. Having this in mind, the addition of krilium at amounts of 1.0 g kg^{-1} or higher clearly increased the stability of the aggregates. Kozak *et al.* (7) showed that in a particle size distribution when relatively small mass is associated to large fractions, the value of D is greater than 3, however, value of D smaller than 3 may be obtained when relatively large mass is associated to large fractions. For such distribution, the corresponding D can not be considered as a parameter of fractal fragmentation according to Turcotte (25) model. In this case, D is merely a fitting parameter.

Fragmentation fractal dimentions obtained from aggregate-size distribution have been correlated to the breakdown of individual aggregates (16), and are shown to be sensitive to soil structural conditions as affected by cropping sequences and tillage methods (6, 20, 21).

Relationships between the D_n and the MWD and D_m obtained by regression analysis are shown in Table 4. The relationship between D_n and MWD for sample treated with petroleum mulches was different from those treated with krilium which is a more effective mulch for aggregate stability. Intercept values of the relationship for all samples treated with petroleum mulches and for krilium treated samples were 4.55 and 4.16, respectively. These values are in agreement with results reported by Perfect and Kay (16) for a silt loam soil. The slopes were -2.32 and -0.99 for petroleum mulches and krilium treated samples, respectively. These values are smaller than those reported by Perfect and Kay (16) for a silt loam soil, suggesting that the model for prediction of D_n may vary with different types of mulch. The relationships (intercepts and slopes) between the D_m and MWD of samples treated with petroleum mulches (except anionic) were not different from those of krilium (Table 4).

For all samples treated with petroleum mulches, the model for D_n and MWD explained 83% of the total variations, indicating that the D_n is equally as good as the MWD (r^2=0.83), and much better than the D_m dimension (r^2=0.42), as a fractal descriptor with appropriate range ($0<D_m<3$). Similar results were reported by Kerfect and Kay (16). For krilium treated samples, the model for Dn and MWD also explained 83% of the total variations, again indicating that the D_n is equally as good as the MWD (r^2=0.83). For all treated samples (except anionic) the model D_m and MWD also explained 78 to 94% of the total variations, indicating that the D_m is equally as good as the MWD.

Table 4. Relationships between indirect number-size fractal demension (D_n, a fitting parameter), mass-fractal dimension (D_m), and mean weight diameter (MWD).

	D_n		D_m	
Type of mulch	Regression equation	R	Regression equation	R
Anionic	D_n=4.42-2.13 (MWD)	-0.93*	†	
Cationic	D_n=4.58-2.18 (MWD)	-0.90*	D_m=2.91-0.28 (MWD)	-0.93*
Clay emulsion	D_n=4.66-2.68 (MWD)	-0.96*	D_m=2.87-0.24 (MWD)	-0.97*
All petroleum mulches	D_n=4.55-2.32 (MWD)	-0.91**	D_m=2.88-0.26 (MWD)	-0.88***
Krilium	D_n=4.16-0.99 (MWD)	-0.91*	D_m=2.79-0.28 (MWD)	-0.96**

*,**,***, Significant at the 0.05, 0.01 and 0.001 probability levels, respectively.

† Correlation was not significant.

For petroleum mulch applications, D_m values were different at the various application rates. Therefore, it was not possible to show the effect of mulch application on the increase of aggregate stability. This might be due to the aggregate size smaller than 0.5 mm as shown by MWD (Table 3). However, for krilium application, D_m values were different at the various application rates similar to those obtained by MWD, but D_m values were similar for application rates of 1.0 and 1.5 g kg^{-1} of krilium, which is not similar to those obtained by MWD. The results of D_n (fitting parameter) showed the differences in the fragment size distributions at the various application rates of petroleum mulches more precisely than the MWD for different krilium application rates.

The direct estimate of the fractal dimension D from mechanical sieving data was proposed by Turcotte (25), Tyler and Wheatcraft (27, 28) and Perfect and Kay (16). The number based on fitting parameter (D_n) was reported to be sensitive to the selected size fractions and scale invariant particle density assumption (28). Therefore, the estimation of fractal dimension (fitting parameter) based on indirect determination of particle number through apparent bulk density of aggregates, may result in values of D_n greater than 3.0, as reported by Tyler and Wheatcraft (27). The values of D greater than 3.0 have also been reported by Rasiah and Biederbeck (21) and Rasiah *et al.* (22) due to linear fitting procedure. However, the D_n values for krilium with application amounts of greater than 0.5 g kg^{-1} were smaller than 3.0 and were also in agreement with D_m (Table 3). According to the equations in Table 4 the MWD for D_n values (a fitting parameter) being appropriate for aggregate stability analysis are 0.67 and 1.17 for petroleum mulches and krilium applications, respectively. Furthermore, the estimation of D_n, based on direct determination of particle number, resulted in values of D_n smaller than 3.0 which were in agreement with those obtained by mass-size distribution analyses (D_m) for a particle diameter range of 0.5 to 32 mm (19). It was shown by Crawford *et al.* (5) that for the fragmented aggregates, the value of D_n may be in the range of $0<D_n<3$. This may occur for the aggregates with diameters greater than 0.5 mm as shown in Table 3 and by Perfect *et al.* (19). Crawford *et al.*(5) also indicated that for incompletely fragmented aggregates, the fractal dimension (fitting parameter) may be overestimated (>3.0) if the fragmentation probability increases with aggregate diameter. Therefore, it may be concluded that application of krilium at amounts of greater than 0.5 g kg^{-1} resulted in aggregates with complete fragmentation properties. Several possible explanations of D>3 were also offerred by McBratney (9), Rasiah *et al.* (20) and Kozak *et al.* (7) .

CONCLUSIONS

Application of MWD, D_n and D_m may lead to different conclusions in the assessment of the effectiveness of various chemicals for improving soil aggregate stability. For less effective chemicals (petroleum mulches) D_n values (a fitting parameter) were a better indicator of differences in fragment distribution, while for more effective chemicals (krilium) MWD showed the differences in fragment-size distributions better than D_m and D_n.

The D_n (a fitting parameter) decreased with increased application amount of mulches. The decrease was most pronounced and consistent for krilium which is known as an effective aggregate stabilizer. The smaller value of D_n indicated that more large aggregates were present in the sample being analyzed showing greater effectiveness of the mulch and of the amount of application. The analyses of variance of D_n showed smaller variability (smaller coefficient of variation) than was obtained with D_m or MWD. The D_n parameter was less sensitive to type of mulch than MWD and D_m but along with MWD, it was more sensitive to application amount than D_m. The D_n values were well correlated with MWD values for all types of soil conditioners but the slope of regression equation was much smaller for krilium than those of other mulches. The D_m values were well correlated with MWD values for applied conditioners except anionic emulsion, and the coefficient of regression equations were similar. Furthermore, it was concluded that the application of krilium with amounts of greater than 0.5 g kg^{-1} resulted in aggregates with complete fragmentation properties, while the aggregates of other treatments did not show this behavior. The results also indicated that the D_n (a fitting parameter) may be used to describe fractal behavior of the soil aggregates with MWD of 0.67 and 1.17 mm for petroleum mulch and krilium applications, respectively. However, D_m failed to show the effects of different rates of petroleum mulches on the fragment size distributions.

LITERATURE CITED

1. **Baldock, J.A. and B.D. Kay. 1987. Influence of cropping history and chemical treatments on the water-stable aggregates of a silt loam soil. Can. J. Soil Sci. 67:501-511.**
2. **Bartoli, F., R. Philippy, M. Doirisse, S. Niquet and M. Dubuit. 1991. Structure and self-similarity in silty and sandy soils: The fractal approach. J. Soil Sci. 42:167-185.**
3. **Brakensiek, D.L. and W.J. Rawls. 1992. Comments on "Fractal processes in soil water retention" by S.W. Tyler and S.W. wheatcraft. Water Resour. Res. 28:601-602.**

4. Brakensiek, D.L, W.J. Rawls, S.D. Logsdon and M. Edwards. 1992. Fractal description of macroporosity. Soil Sci. Soc. Amer. J. 56:1721-1723.
5. Crawford, J.W., B.D. Sleeman and I.M. Young. 1993. On the relation between number-size distributions and the fractal dimension of aggregates. J. Soil Sci. 44:555-565.
6. Eghball, B., L.N. Mielke, G.A. Calvo and W.W. Wilhelm. 1993. Fractal description of soil fragmentation for various tillage methods and crop sequences. Soil Sci. Soc. Amer. J. 57:1337-1341.
7. Kozak, Ya. E., A. Pachepsky, S. Sokolowski, Z. Sokolowska and W. Stepniewski. 1996. A modified number-based method for estimating fragmentation fractal dimentions of soils. Soil Sci. Soc. Amer. J. 60:1291-1297.
8. Logsdon, S.D. 1995. Analysis of aggregate fractal dimensions and aggregate densities back-calculated from hydraulic conductivity. Soil Sci. Soc. Amer. J. 59:1216-1221.
9. McBratney, A.B. 1993. Comments to "Fractal dimensions of soil aggregate-size distributions calculated by number and mass". Soil Sci. Soc. Amer. J. 57:1393-1394.
10. Mandelbrot, B.B. 1982. The Fractal Geometry of Nature. W.H. Freemân, San Francisco, CA, U.S.A.
11. Mazurak, A.P. 1950. Effect of gaseous phase on water-stable synthetic aggregates. Soil Sci. 69:135-148.
12. Moosavi, S.A.A. and A.R. Sepaskhah. 1983. Seedling emergence and growth of sugarbeet as influenced by surface and incroporation applications of petroleum mulches. Iran Agric. Res. 2:23-37.
13. Mossavi, S.A.A. and A.R. Sepaskhah. 1983. Effects of Iranian petroleum mulches on some physical properties of a calcareous soil. I. Aggregate stability and water holding capacity. Iran Agric. Res. 2:147-154.
14. Niemeyer, J. and C. Ahl. 1991. Die Fractal Verteilung der Minerale und deren Bedeutung fur die sorption. Mitt. Dtsch. Bodenkd. Ges. 66:217-274.
15. Pachepsky, Y.A. A.T. Polubesova, M. Hajnos, Z. Sokolowska and G. Josefaciuk. 1995. Fractal parameters of pore surface area as influenced by simulated soil degradation. Soil Sci. Soc. Amer. J. 59:68-75.

16. Perfect, E. and B.D. Kay. 1991. Fractal theory applied to soil aggregation. Soil Sci. Soc. Amer. J. 55:1552-1558.
17. Perfect, E. and B.D. Kay. 1995. Brittle fracture of fractal cubic aggregates. Soil Sci. Soc. Amer. J. 59:969-974.
18. Perfect, E., B.D. Kay and V. Rasiah. 1993. Multifractal model for soil aggregate fragmentation. Soil Sci. Soc. Amer. J. 57:896-900.
19. Perfect, E., V. Rasiah and B.D. Kay. 1992. Fractal dimensions of soil aggregate-size distributions calculated by number and mass. Soil Sci. Soc. Amer. J. 56:1407-1409.
20. Rasiah, V., B.D. Kay, and E. Perfect. 1993. New mass-based model for estimating fractal dimensions of soil aggregates. Soil Sci. Soc. Amer. J. 57:891-895.
21. Rasiah, V. and V.O. Biederbeck. 1995. Fractal dimension of soil aggregates: Influence of bulk density, fitting procedure, and oily waste sludge incorporation. Soil Sci. 160:250-255.
22. Rasiah, V., B.D. Kay, and E. Perfect. 1995. Linear and nonlinear estimates of fractal dimension for soil. Aggregate fragmentation. Soil Sci. Soc. Amer. J. 59:83-87.
23. Rieu, M. and G. Sposito. 1991. Fractal fragmentation, soil porosity, and soil water properties: I. Theory. Soil Sci. Soc. Amer. J. 55:1231-1238.
24. Toledo, P., R.A. Novy, H.T. Davis and L.E. Serivan. 1990. Behavior of hydraulic conductivity at low water contents. Soil Sci. Soc. Amer. J. 54:673-679.
25. Turcotte, D.L. 1986. Fractals and fragmentation. J. Geophys. Res. 91: 921-1926.
26. Turcotte, D.L. 1992. Fractals and Chaos in Geology and Geophysics. Cambridge University Press, Cambridge, England.
27. Tyler, S.W. and S.W. Wheatcraft. 1989. Application of fractal mathematics to soil water retention estimation. Soil Sci. Soc. Amer. J. 53:987-996.
28. Tyler, S.W. and S.W. Wheatcraft. 1992. Fractal scaling of soil particle-size distribution: Analysis and limitations. Soil Sci. Soc. Amer. J. 56:362-369.

29. Van Bavel, C.H.M. 1949. Mean weight-diameter of soil aggregates as a statistical index of aggregation. Soil Sci. Soc. Amer. Proc. 14:20-23.

30. Van Bavel, C.H.M. 1953. Report of the committee on physical analyses 1951-1953. Soil Sci. Soc. Amer. Proc. 17:416-418.

31. Wu, Q., M. Borkovec, and H. Sticher. 1993. On particle-size distributions in soils. Soil Sci. Soc. Amer. J. 57:883-890.

32. Young, I.M. and J.W. Crawford. 1991. The fractal structure of soil aggregates: its measurement and interpretation. J. Soil Sci. 42:187-192.